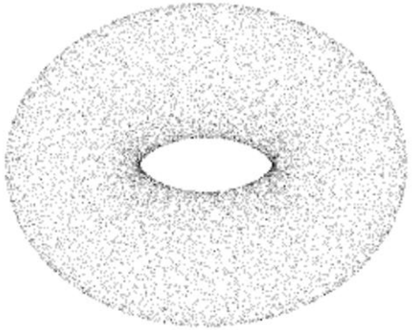
3D Scanning & Motion Capture

Surface Representations

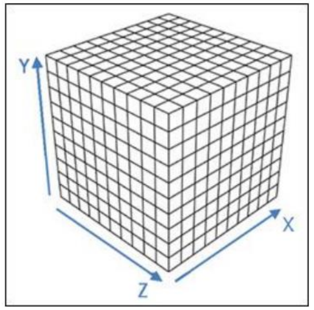
Prof. Matthias Nießner

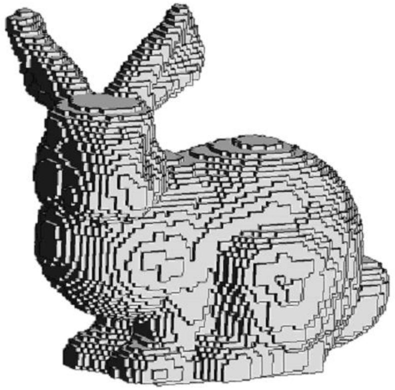
Last Lecture: What is “3D”?

- Point Clouds

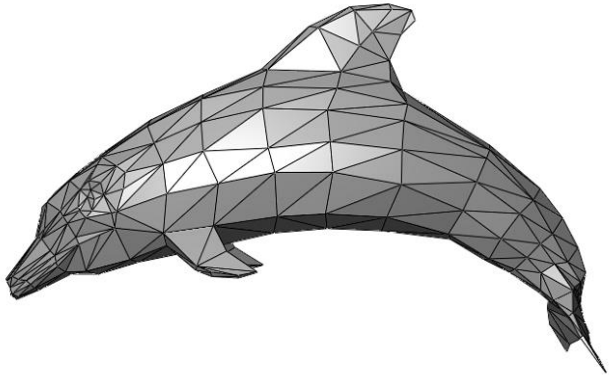


- Voxels

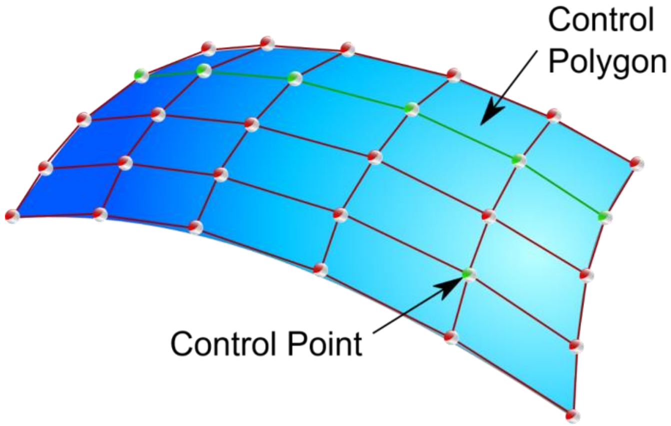




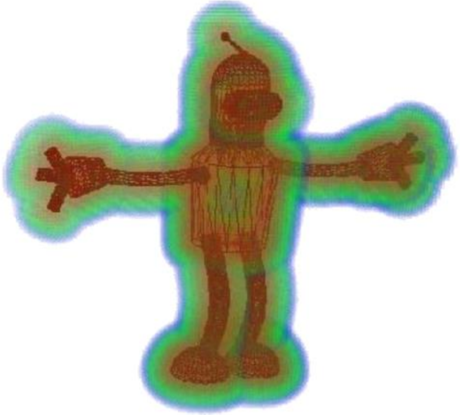
- Polygonal Meshes

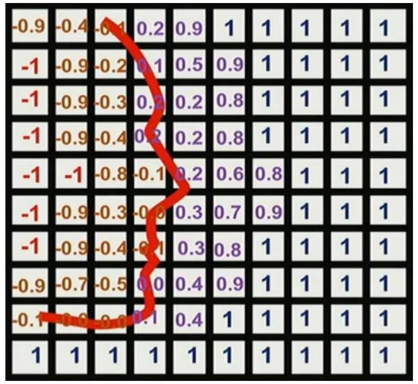


- Parametric Surfaces



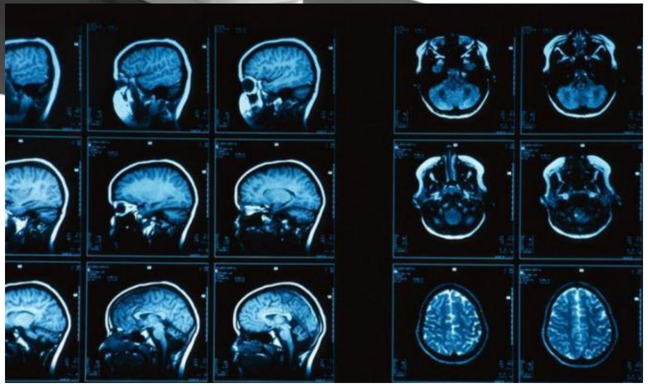
- Implicit Surfaces

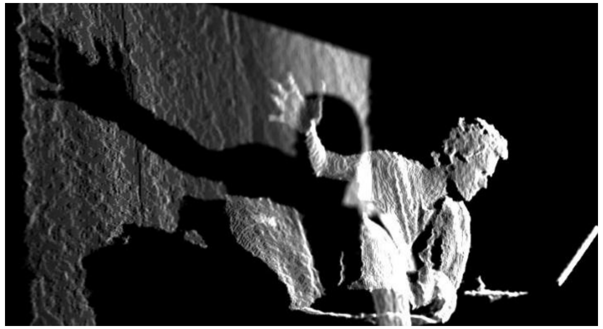


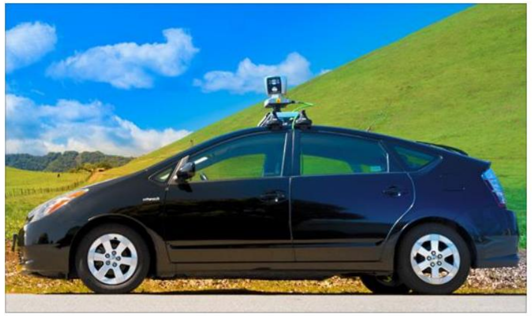


Truncated Signed Distance Field (TSDF)

Last Lecture: How to obtain “3D”?



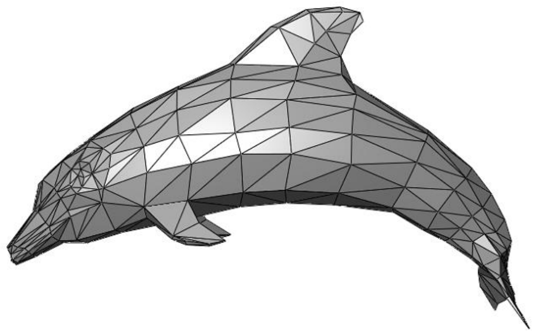




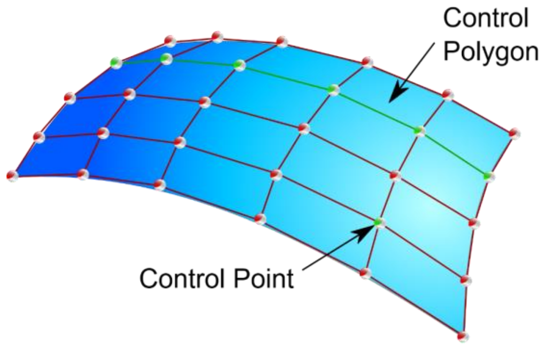
Today: Surface Representations

Overview

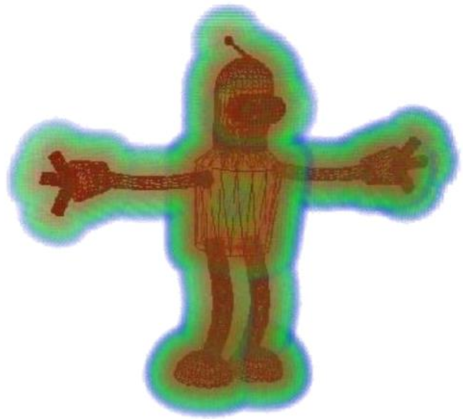
- Polygonal Meshes



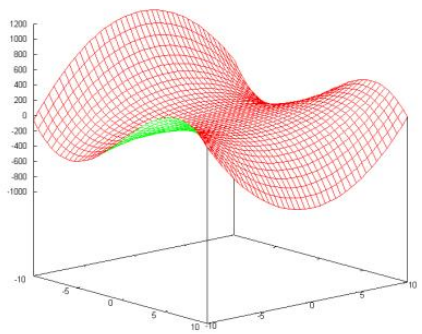
- Parametric Surfaces



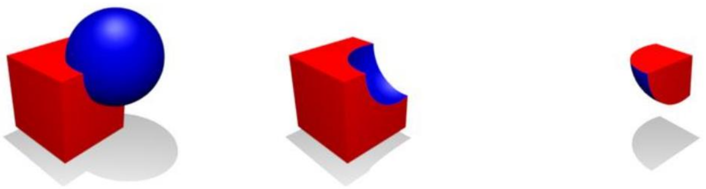
- Implicit Surfaces



- Explicit Surfaces

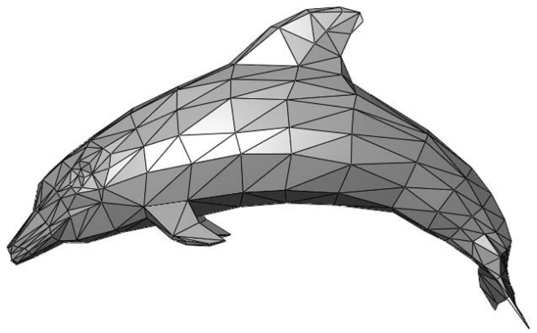


- Constructive Solid Geometry

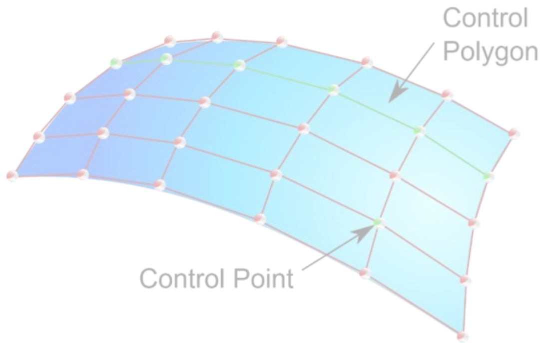


Overview

- Polygonal Meshes

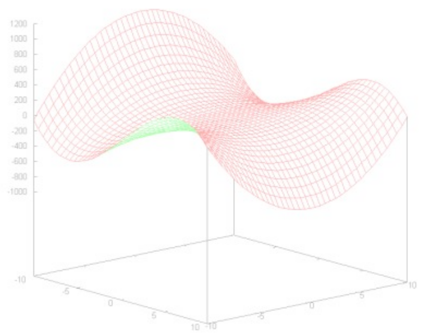


- Parametric Surfaces



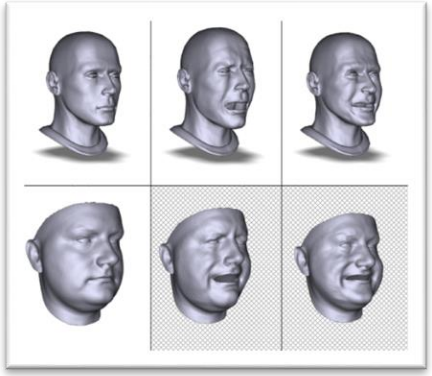
- Implicit Surfaces

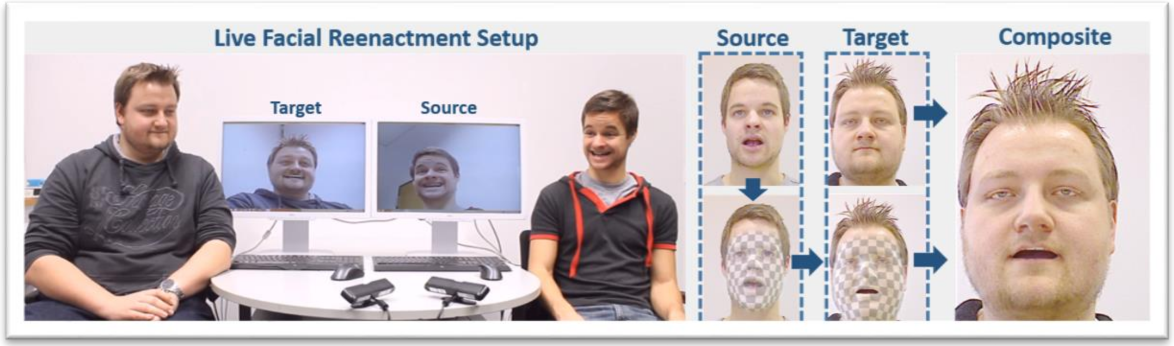
- Explicit Surfaces

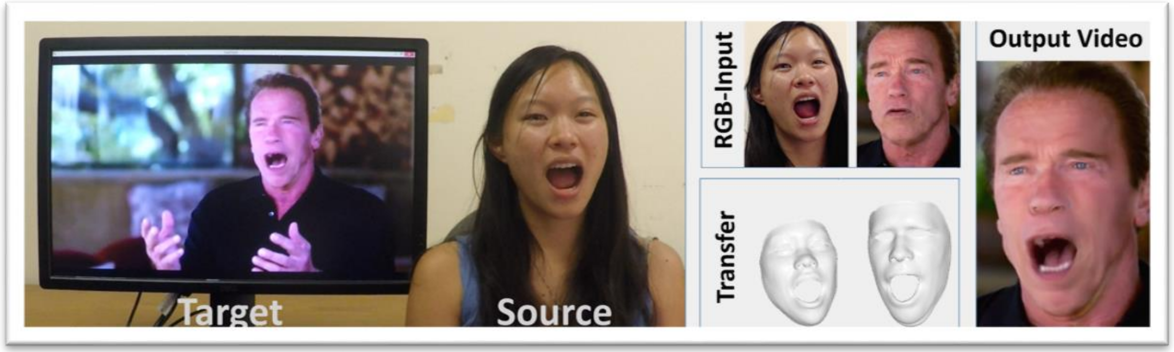


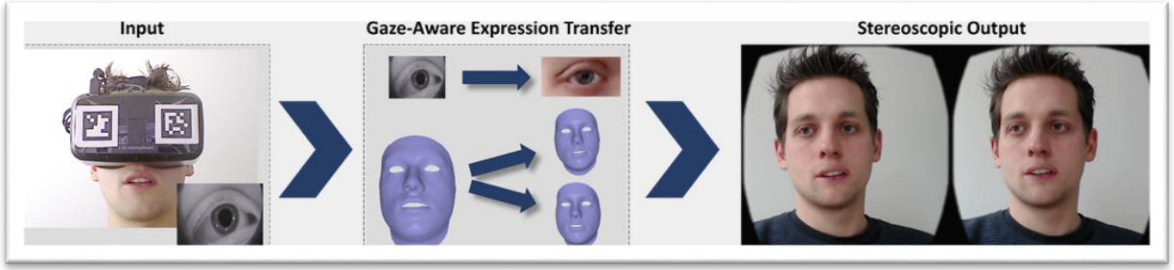
- Constructive Solid Geometry

Polygonal Meshes



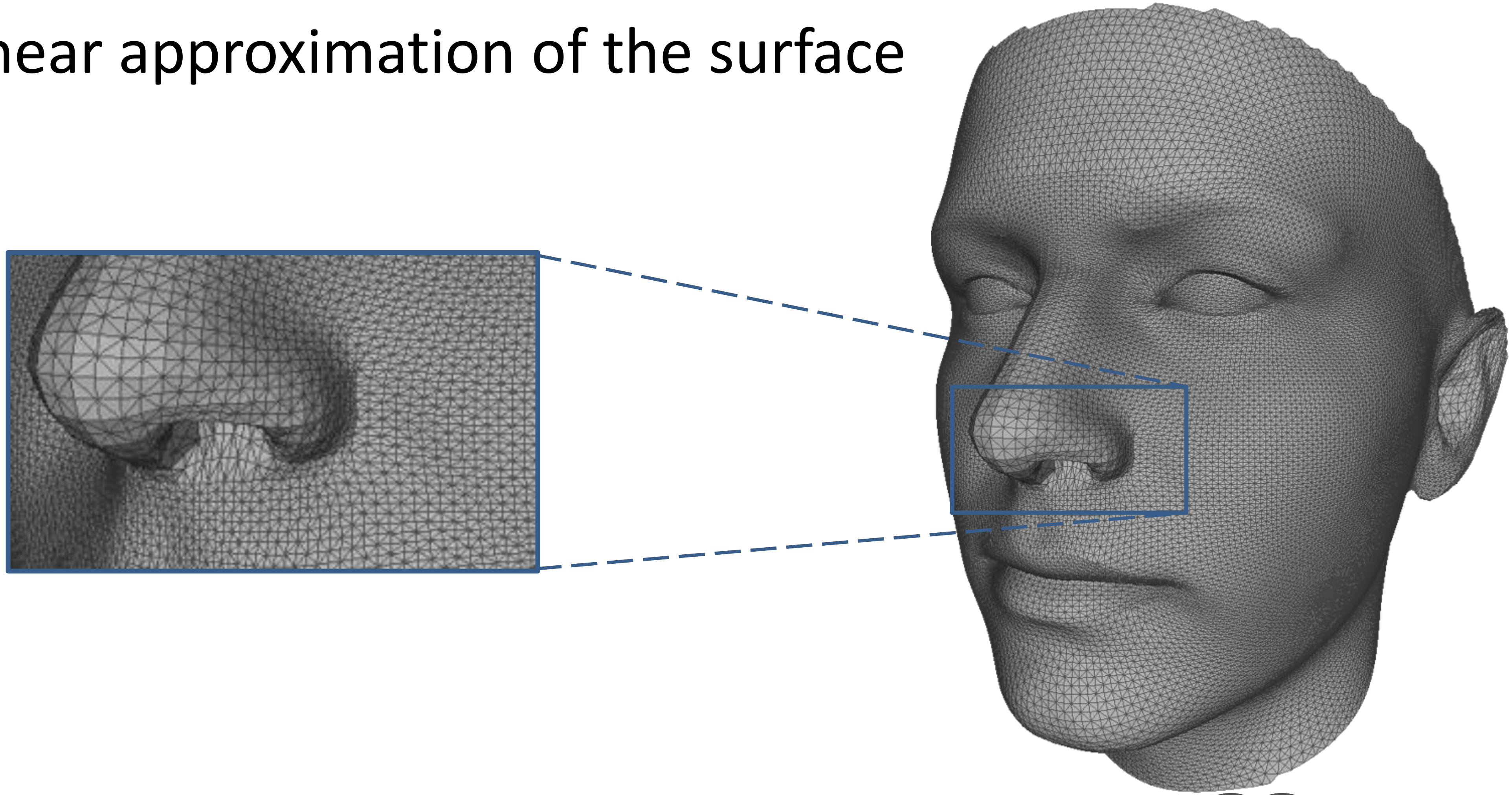






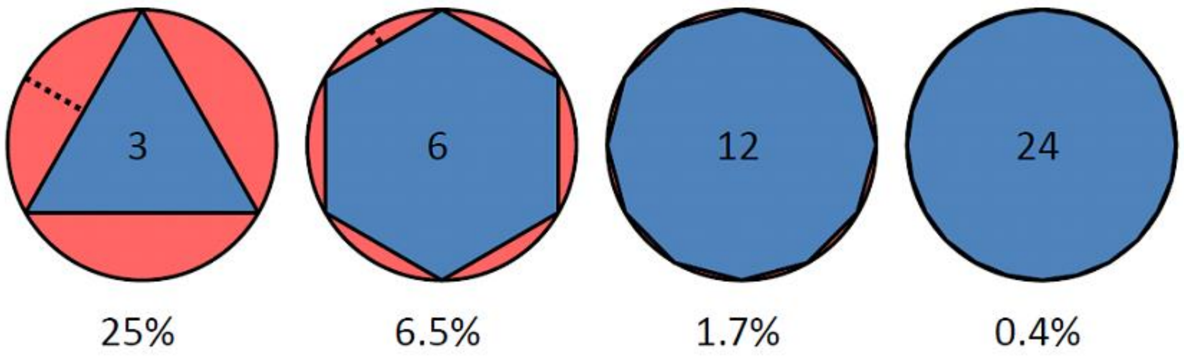
Polygonal Meshes

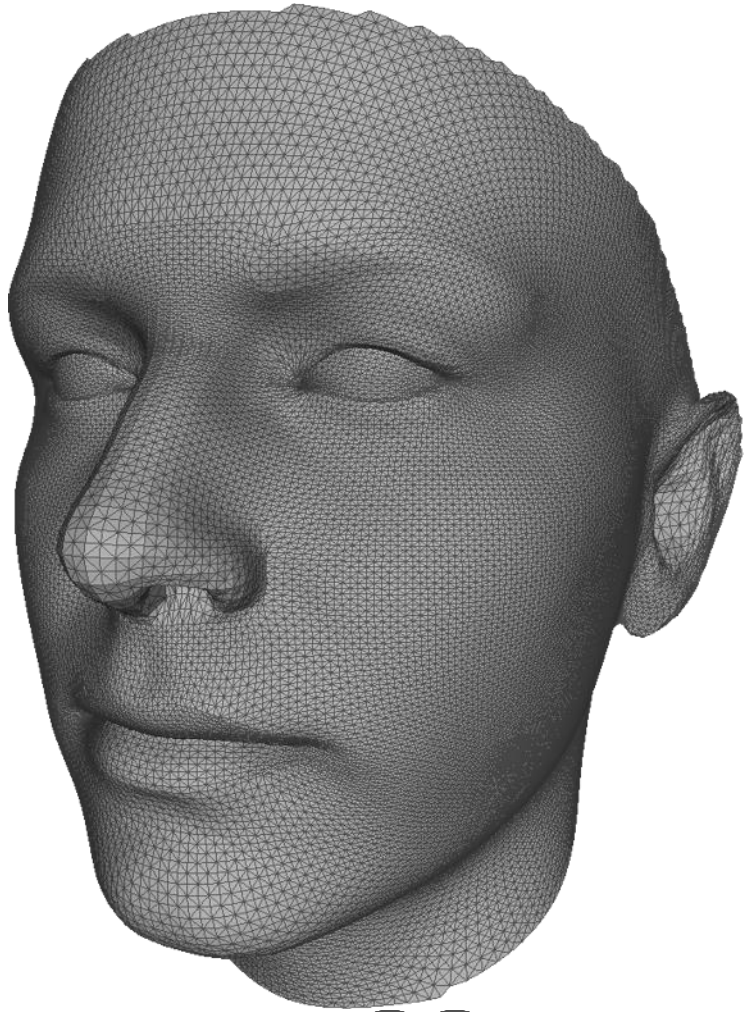
- Piecewise linear approximation of the surface



Polygonal Meshes

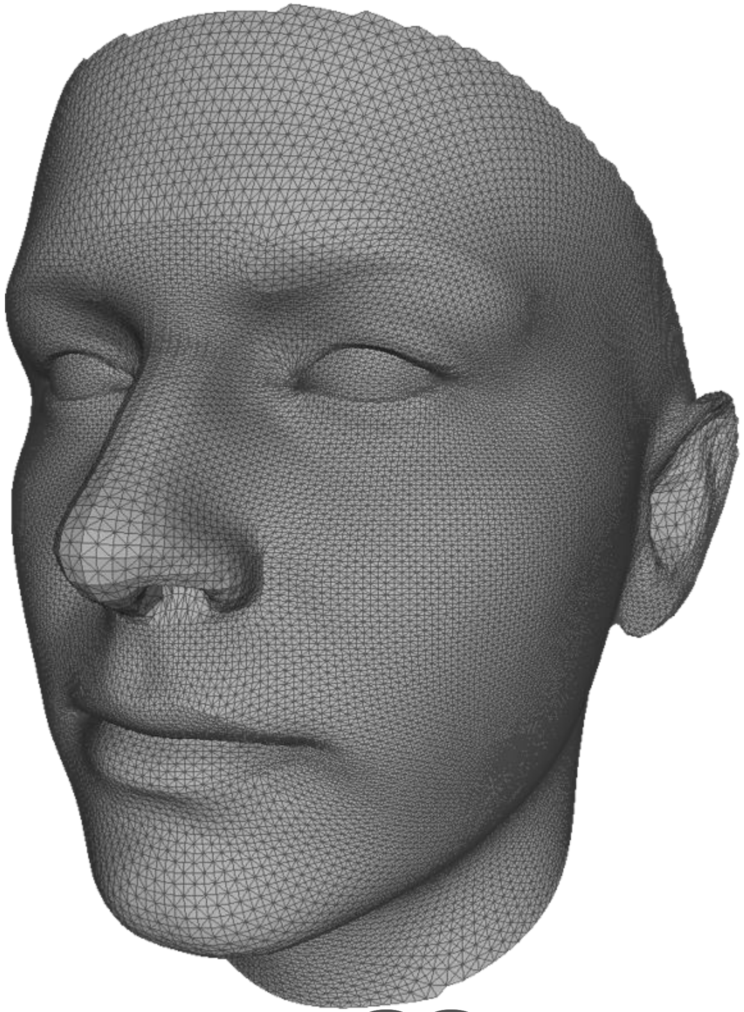
- Piecewise linear approximation of the surface
  - Error $O(h^2)$





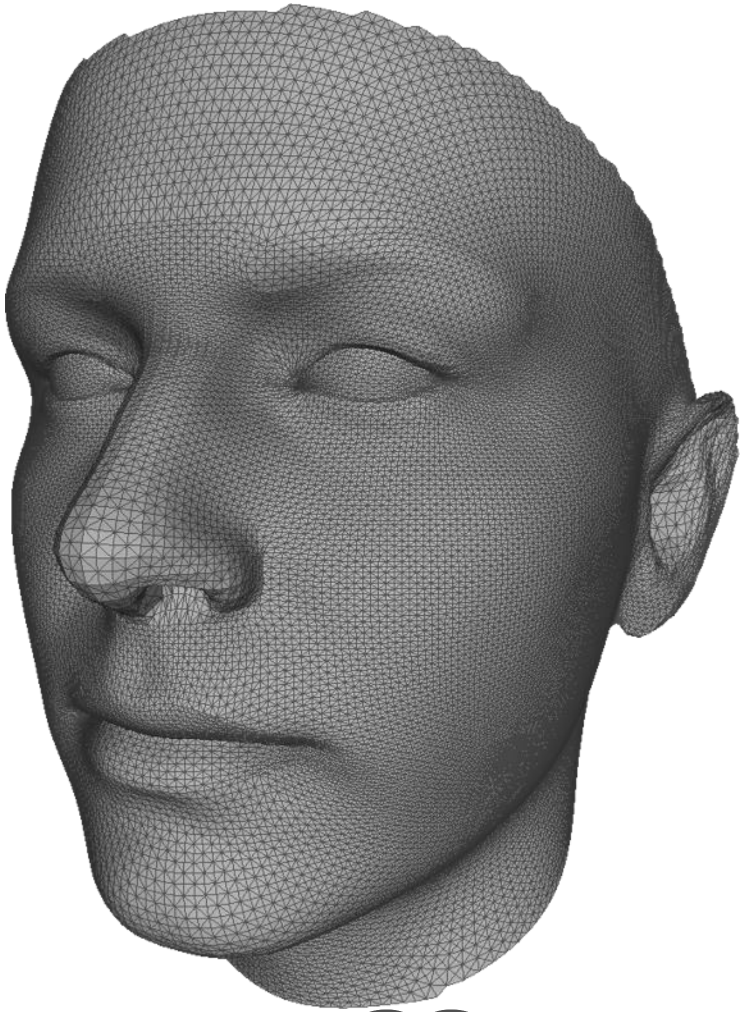
Polygonal Meshes

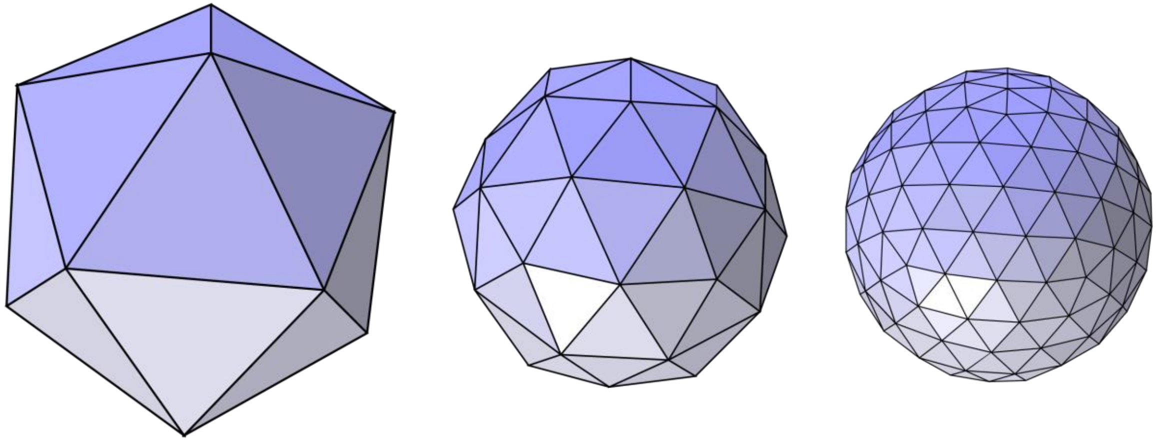
- Piecewise linear approximation of the surface
  - Error $O(h^2)$
  - Arbitrary topology



Polygonal Meshes

- Piecewise linear approximation of the surface
  - Error $O(h^2)$
  - Arbitrary topology
  - Adaptive refinement (subdivision)



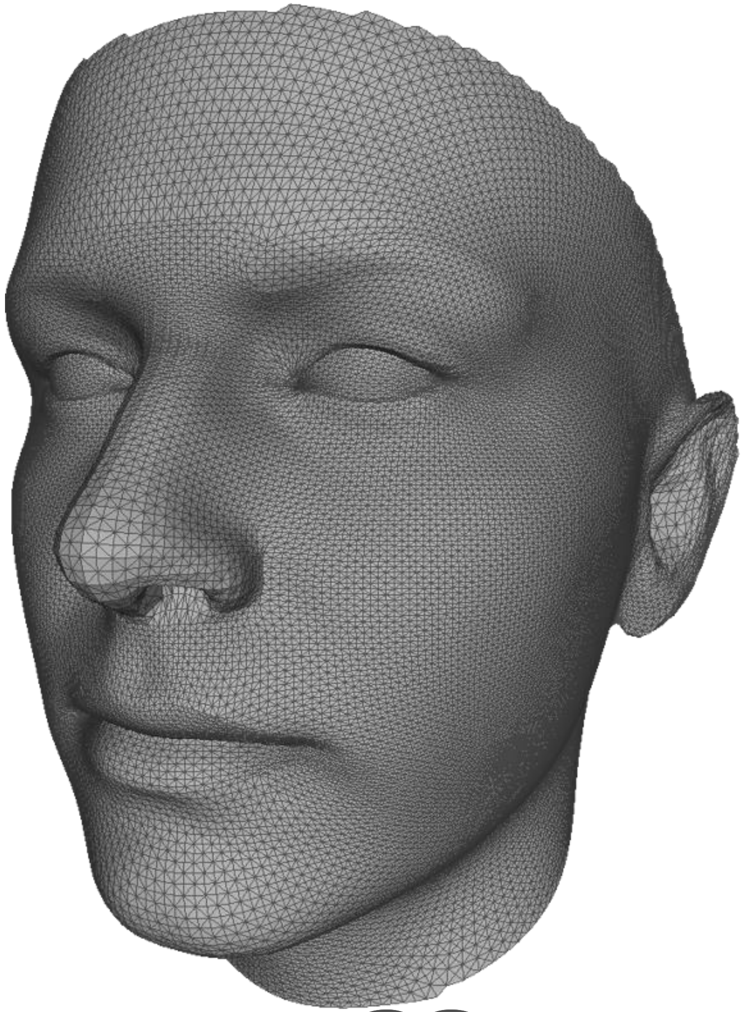


Loop Subdivision Surface

Polygonal Meshes

- Piecewise linear approximation of the surface
  - Error $O(h^2)$
  - Arbitrary topology
  - Adaptive refinement (subdivision)
  - Efficient rendering

Rasterization on modern GPUs is darn fast!
E.g., my old 1080 GTX rasters 2-3 mio polygons in 0.2-0.3ms (including textures, etc.)

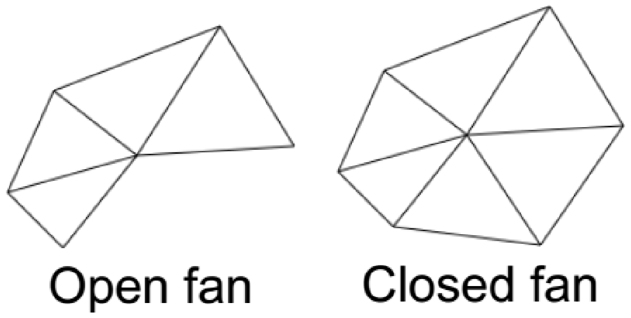


Polygonal Meshes

- A polygonal mesh is a collection of polygons satisfying certain restrictions

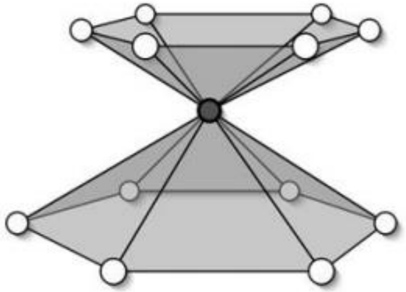
Polygonal Meshes

- A triangle mesh is called **manifold** if:
  - The intersection of two triangles is:
    - Empty
    - A common vertex
    - A common edge
  - Edges have
    - One adjacent triangle → border edge
    - Two adjacent triangles → inner edge
  - For a vertex the adjacent triangles
    - Build a single open fan → border vertex
    - Build a single closed fan → inner vertex

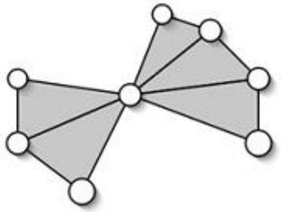


Polygonal Meshes

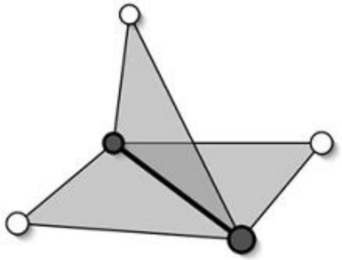
- Example of **non-manifold** meshes:



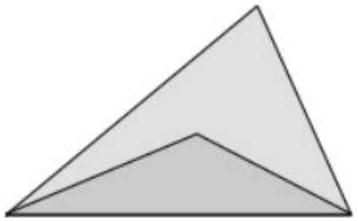
Two closed fans
at one vertex



Two open fans
at one vertex



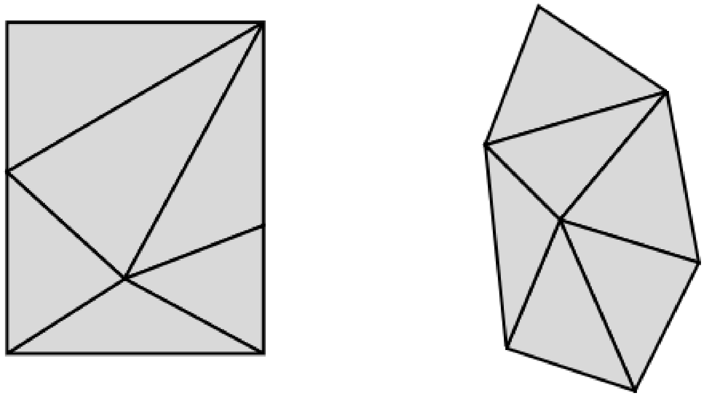
Three triangles
At one edge



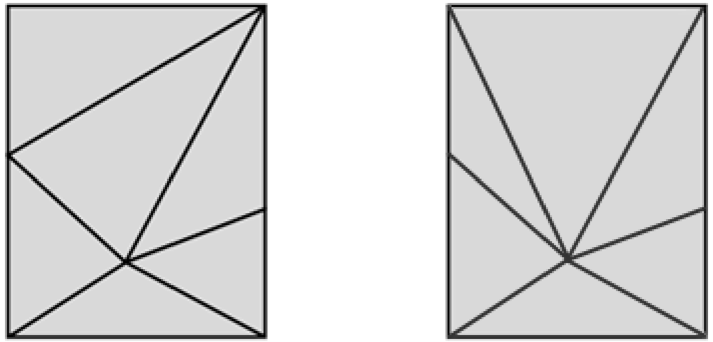
Intersection of triangles
Is an entire triangle

Polygonal Meshes

- Topology vs. Geometry
  - Topologically equivalent:
    - But different geometry



  - Geometrically equivalent:
    - But different topology

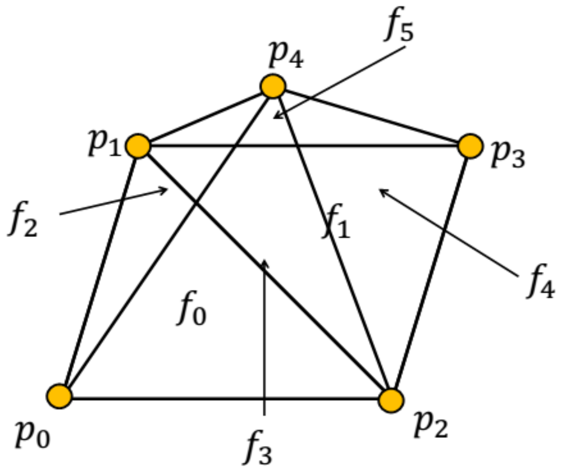


Polygonal Meshes

- Shared Vertex Data Structure
  - De facto standard for file formats (e.g., OFF, OBJ, STL, PLY, …)

vertex list
p_0: x_0, y_0, z_0;
p_1: x_1, y_1, z_1;
p_2: x_2, y_2, z_2;
p_3: x_3, y_3, z_3;
p_4: x_4, y_4, z_4;

face list
f_0: 0, 1, 2;
f_1: 2, 1, 3;
f_2: 1, 0, 4;
f_3: 4, 0, 2;
f_4: 4, 2, 3;
f_5: 4, 3, 1;



Polygonal Meshes

- Shared Vertex Data Structure
  - Cube.obj

```
v 1.000000 -1.000000 -1.000000
v 1.000000 -1.000000 1.000000
v -1.000000 -1.000000 1.000000
v -1.000000 -1.000000 -1.000000
v 1.000000 1.000000 -0.999999
v 0.999999 1.000000 1.000001
v -1.000000 1.000000 1.000000
v -1.000000 1.000000 -1.000000

f 2 3 4
f 8 7 6
f 5 6 2
f 6 7 3
f 3 7 8
f 1 4 8
f 1 2 4
f 5 8 6
f 1 5 2
f 2 6 3
f 4 3 8
f 5 1 8
```

Polygonal Meshes

- Shared Vertex Data Structure
  - Cube.ply

    ply also support supports binary encoding instead of ASCII

```
ply
format ascii 1.0
element vertex 8
property float x
property float y
property float z
element face 12
property list uchar uint vertex_indices
end_header
-1.000000 -1.000000 1.000000
-1.000000 1.000000 1.000000
-1.000000 1.000000 -1.000000
1.000000 1.000000 1.000000
1.000000 1.000000 -1.000000
1.000000 -1.000000 1.000000
1.000000 -1.000000 -1.000000
-1.000000 -1.000000 -1.000000
3 0 1 2
3 1 3 4
3 3 5 6
3 0 7 5
3 7 2 4
3 5 3 1
3 7 0 2
3 2 1 4
3 4 3 6
3 5 7 6
3 6 7 4
3 0 5 1
```

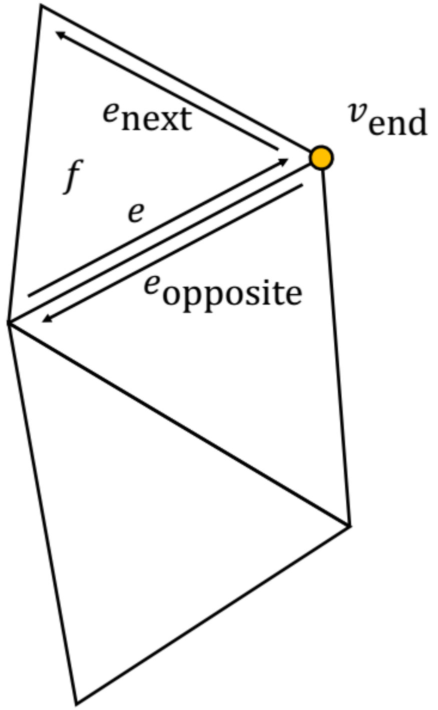
Polygonal Meshes

- Half-Edge Data Structure
  - Easy geometric queries → widely used in geometric computations
  - Only manifolds

Edge
- Edge ID e
- End vertex v_{end}
- Next edge e_{next}
- Opposite edge e_{opposite}
- Face f

Vertex
- Vertex ID
- Index of one incident edge

Face
- Face ID
- Index of one incident edge



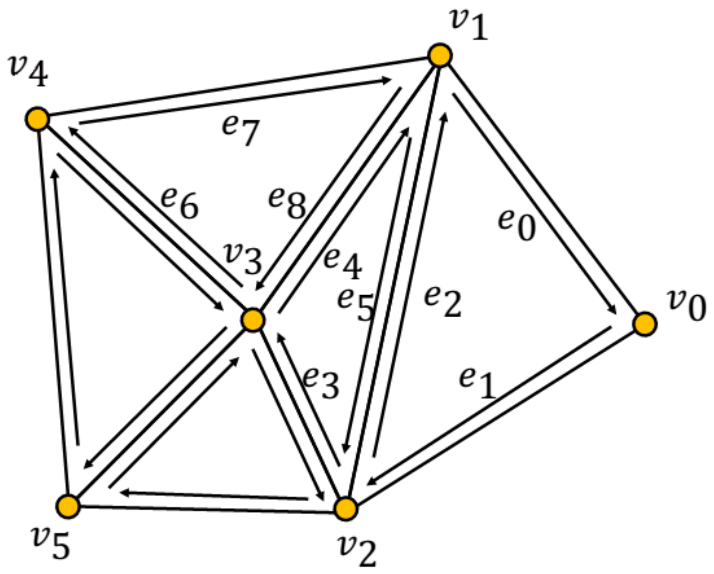
https://www.graphics.rwth-aachen.de/media/openmesh_static/Documentations/OpenMesh-6.1-Documentation/a00016.html

Polygonal Meshes

- Half-Edge Data Structure
  - Easy geometric queries → widely used in geometric computations
  - Only manifolds
  - Triangular Meshes → Directed Edge Data Structure

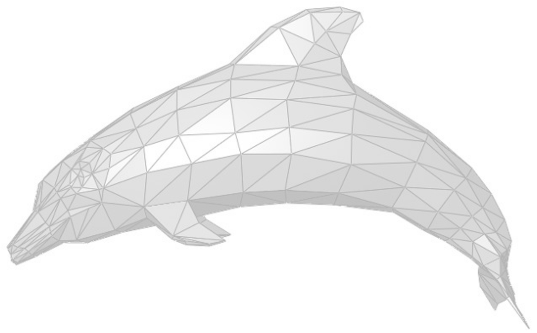
vertex list
p_0: x_0, y_0, z_0;
p_1: x_1, y_1, z_1;
p_2: x_2, y_2, z_2;
p_3: x_3, y_3, z_3;
p_4: x_4, y_4, z_4;
p_5: x_5, y_5, z_5;

edge list
$\left.\begin{array}{l} e_0\text{: 1, -1;} \\ e_1\text{: 0, -1;} \\ e_2\text{: 2, 5;} \end{array}\right\} t_0$
$\left.\begin{array}{l} e_3\text{: 2, X;} \\ e_4\text{: 3, 8;} \\ e_5\text{: 1, 2;} \end{array}\right\} t_1$
e_6: 3, X;
e_7: 4, -1;
…

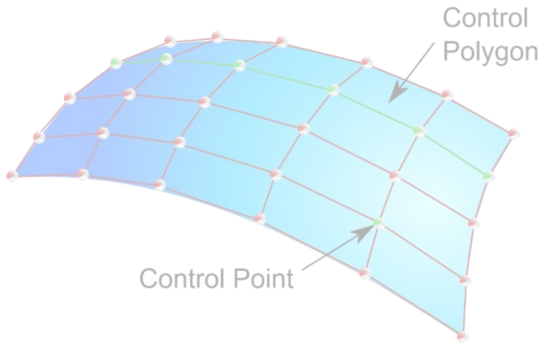


Overview

- Polygonal Meshes

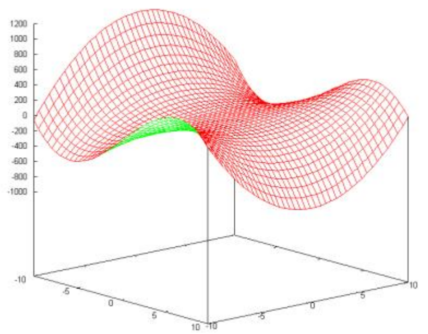


- Parametric Surfaces



- Implicit Surfaces

- Explicit Surfaces

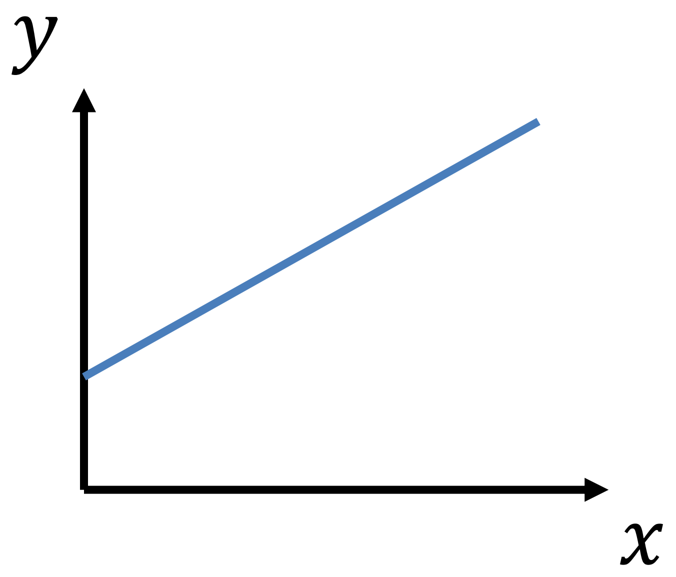


- Constructive Solid Geometry

Explicit Surfaces

- Explicit form:

$$f(x) = y = m \cdot x + c$$



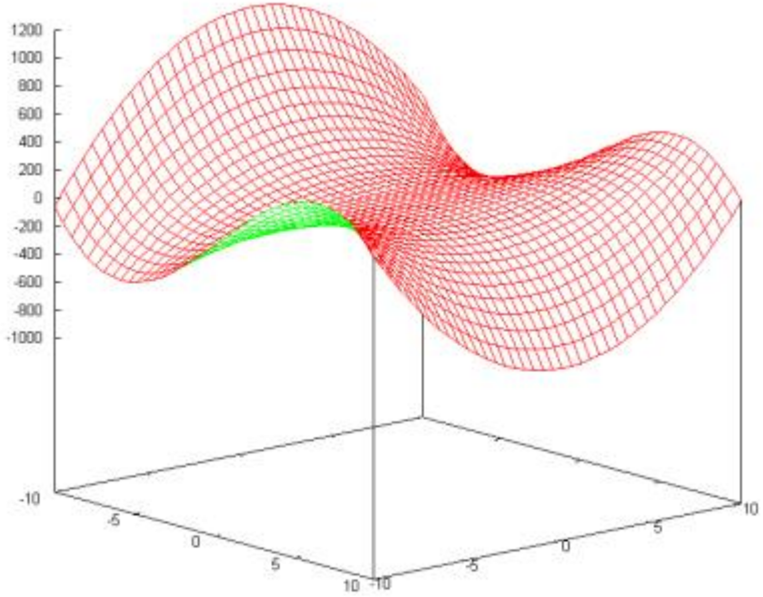
Explicit Surfaces

- Explicit form:

$$f(x, y)\colon\ \mathbb{R}^2 \to \mathbb{R}$$

- Surface is defined by:

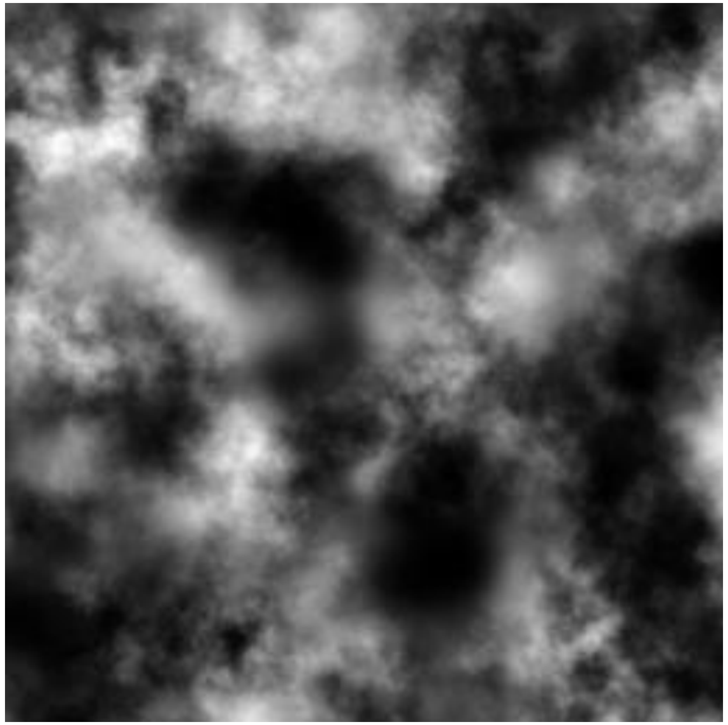
$$S(x, y) = (x, y, f(x, y))$$

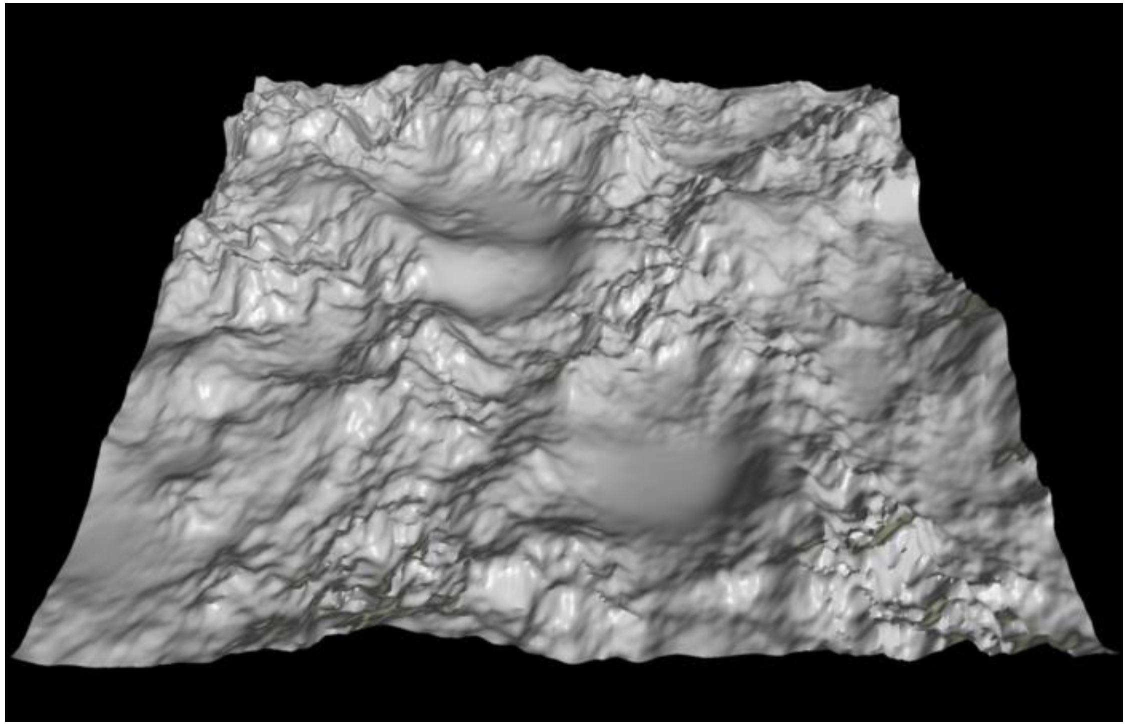


- Disadvantages
  - Restricted shapes, e.g., only one height value per (x,y) pair

Explicit Surfaces

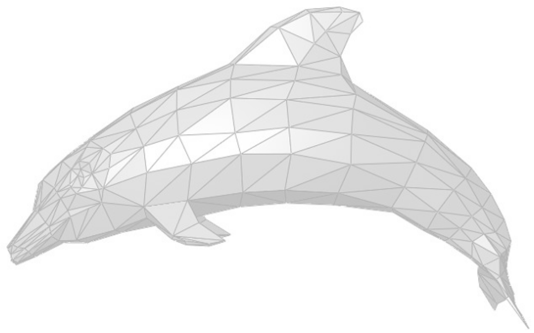
- E.g., height fields



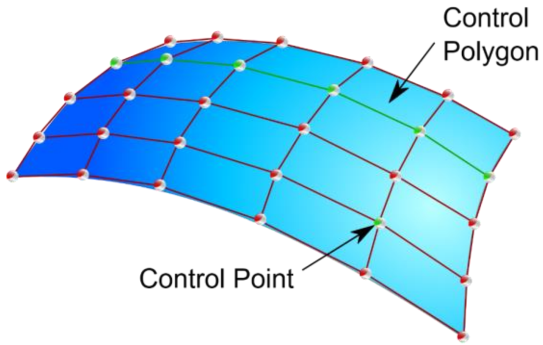


Overview

- Polygonal Meshes

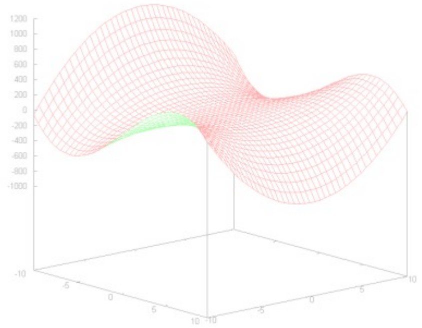


- Parametric Surfaces



- Implicit Surfaces

- Explicit Surfaces



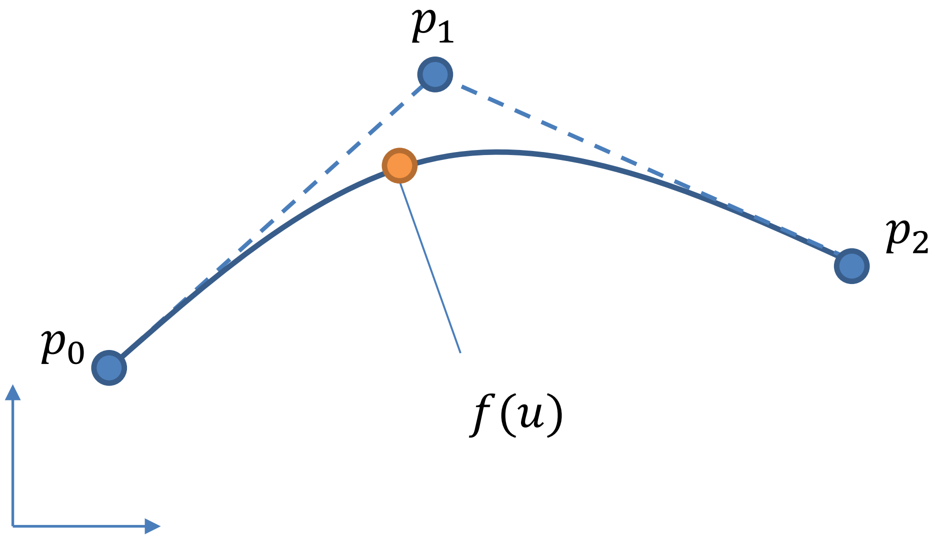
- Constructive Solid Geometry

Parametric Surfaces

– Used in design and construction → freeform surfaces

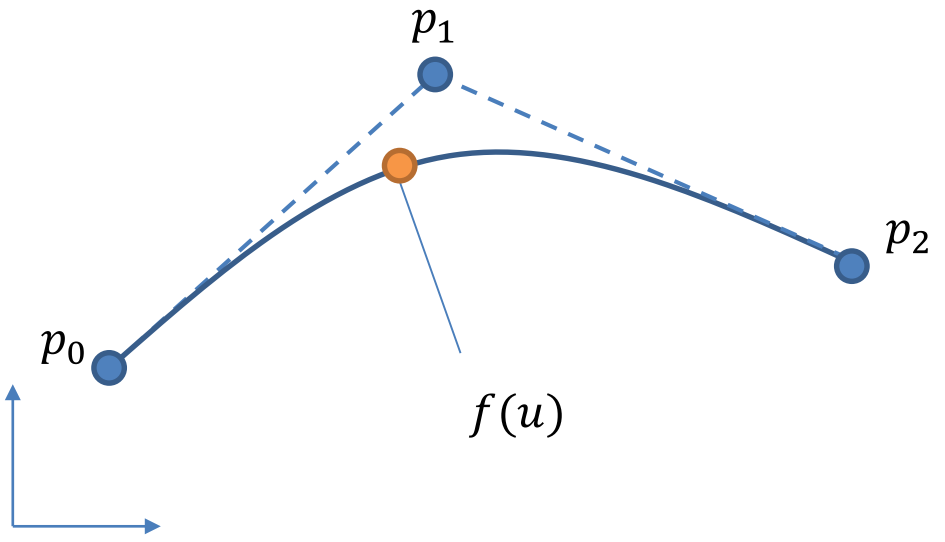
Parametric Surfaces

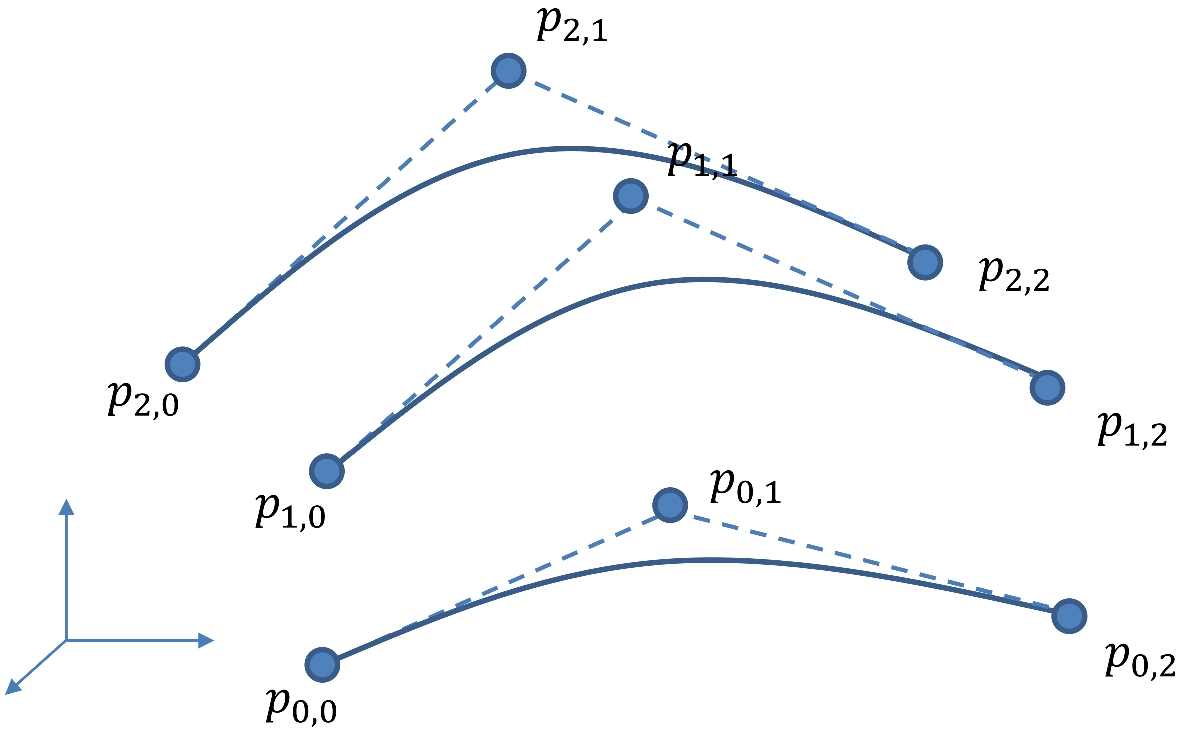
- Example: Bézier Curves & Tensor Product Surface



Parametric Surfaces

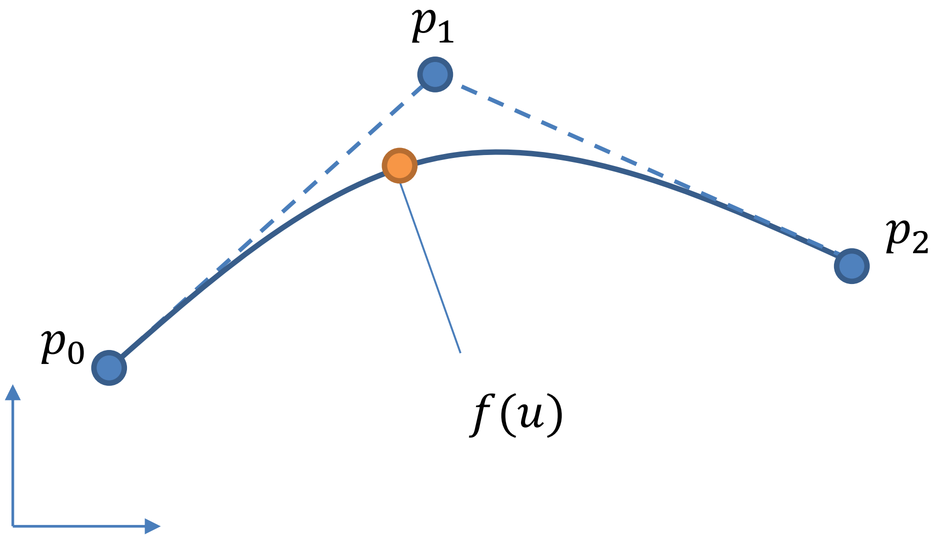
- Example: Bézier Curves & Tensor Product Surface

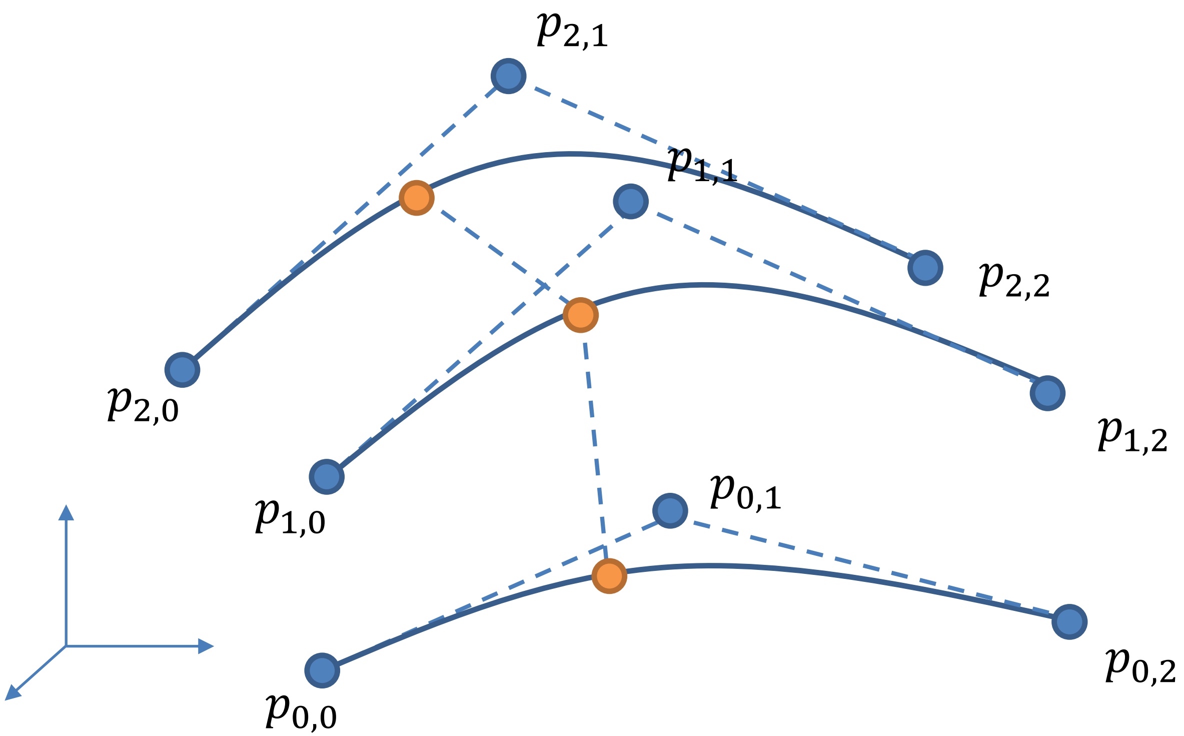




Parametric Surfaces

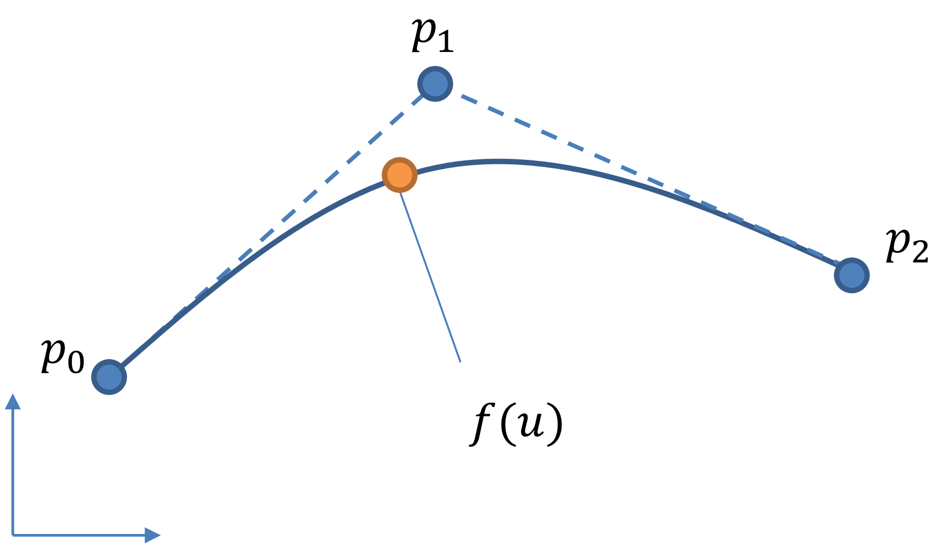
- Example: Bézier Curves & Tensor Product Surface

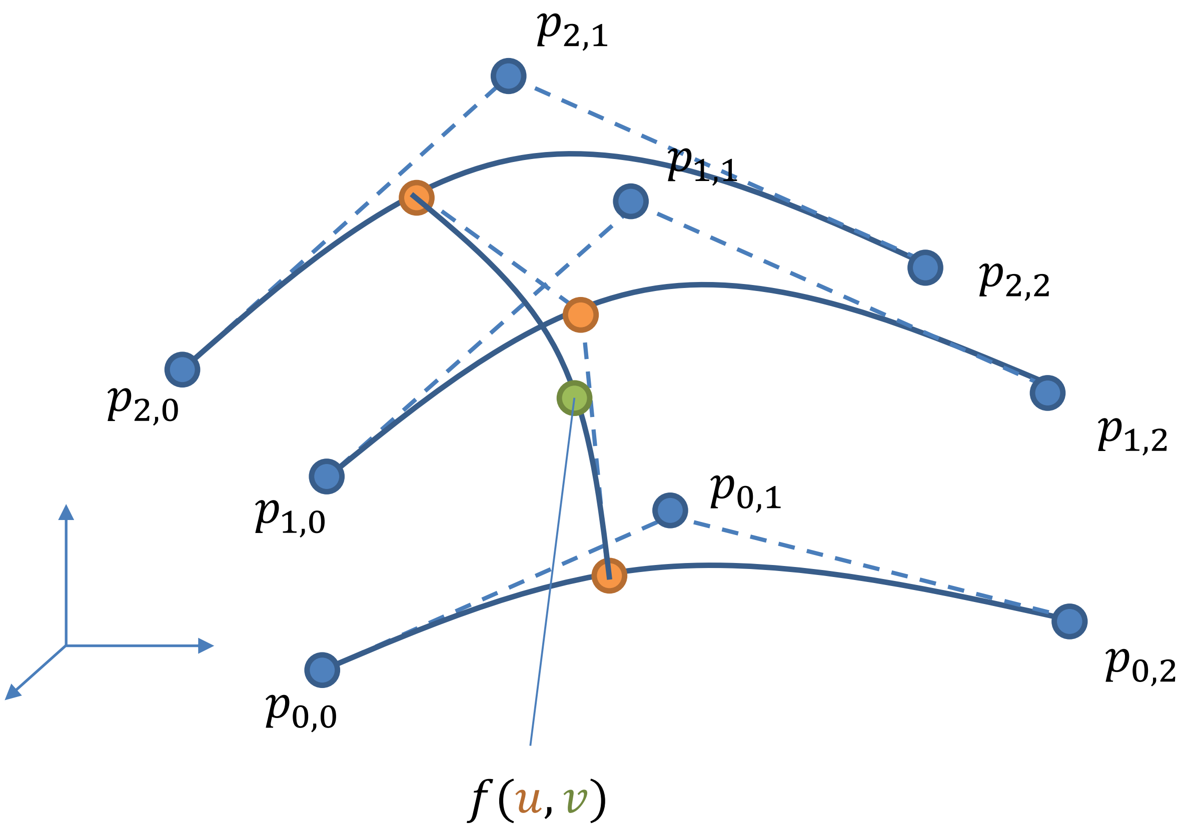




Parametric Surfaces

- Example: Bézier Curves & Tensor Product Surface

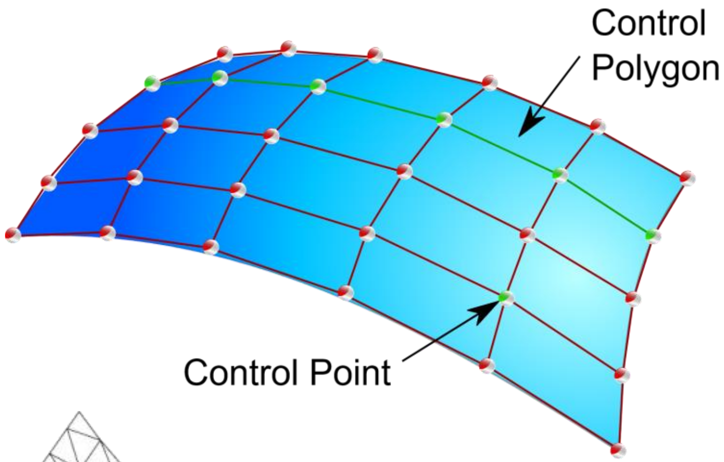




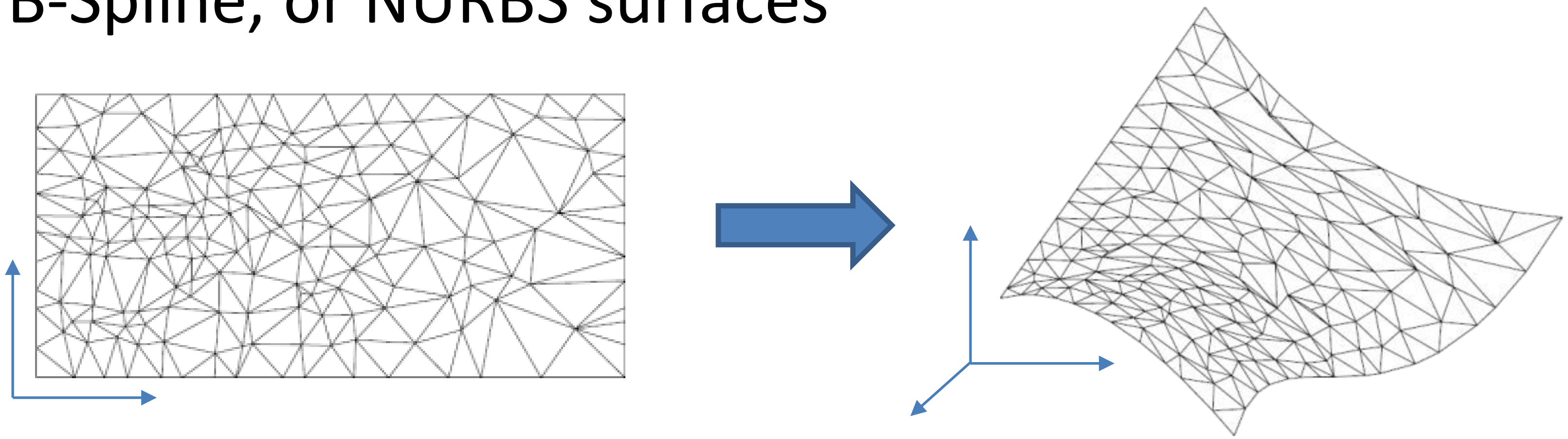
Parametric Surfaces

- Parametric form:

$$f(u, v)\colon \mathbb{R}^2 \rightarrow \mathbb{R}^3$$



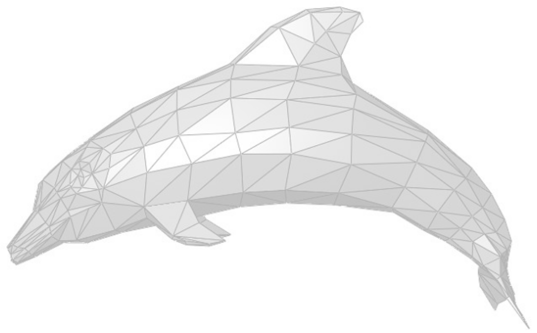
- Maps a bounded 2D domain to 3D
  – Bézier-, B-Spline, or NURBS surfaces



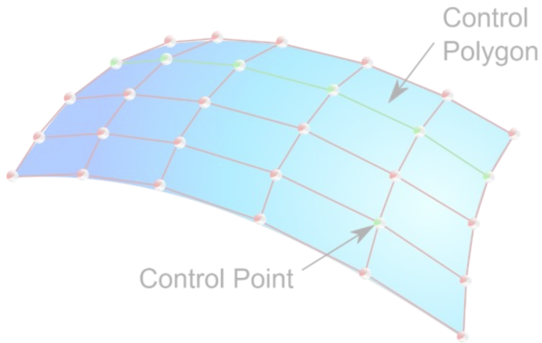
  – Used in design and construction → freeform surfaces

Overview

- Polygonal Meshes

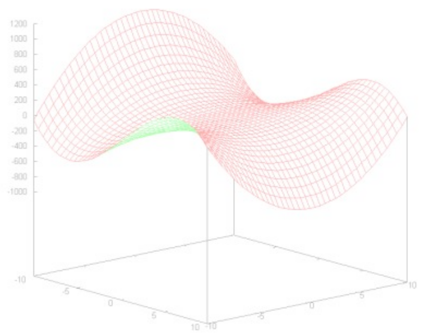


- Parametric Surfaces

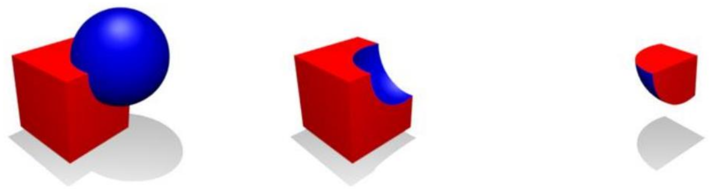


- Implicit Surfaces

- Explicit Surfaces

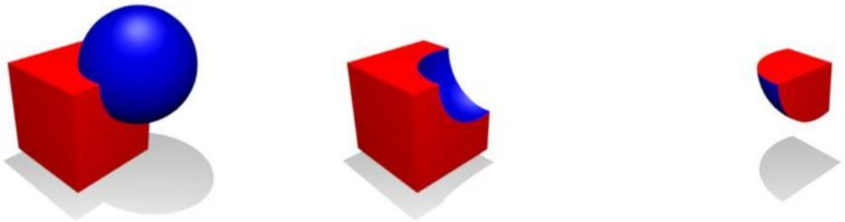


- Constructive Solid Geometry



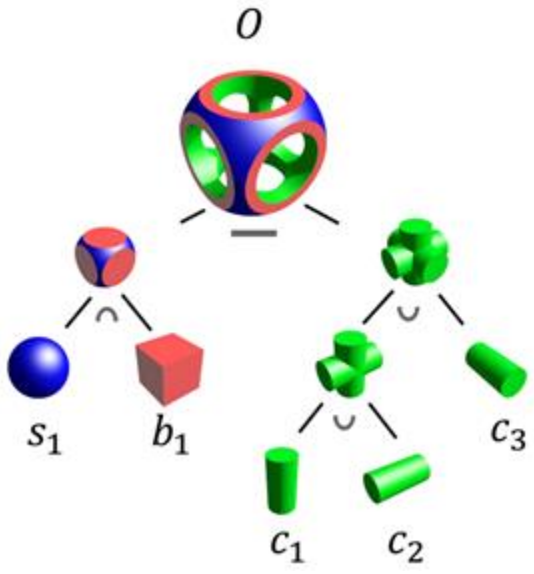
Constructive Solid Geometry

- Surface is defined as the boundary of a solid object that was created by Boolean operations on primitive solids



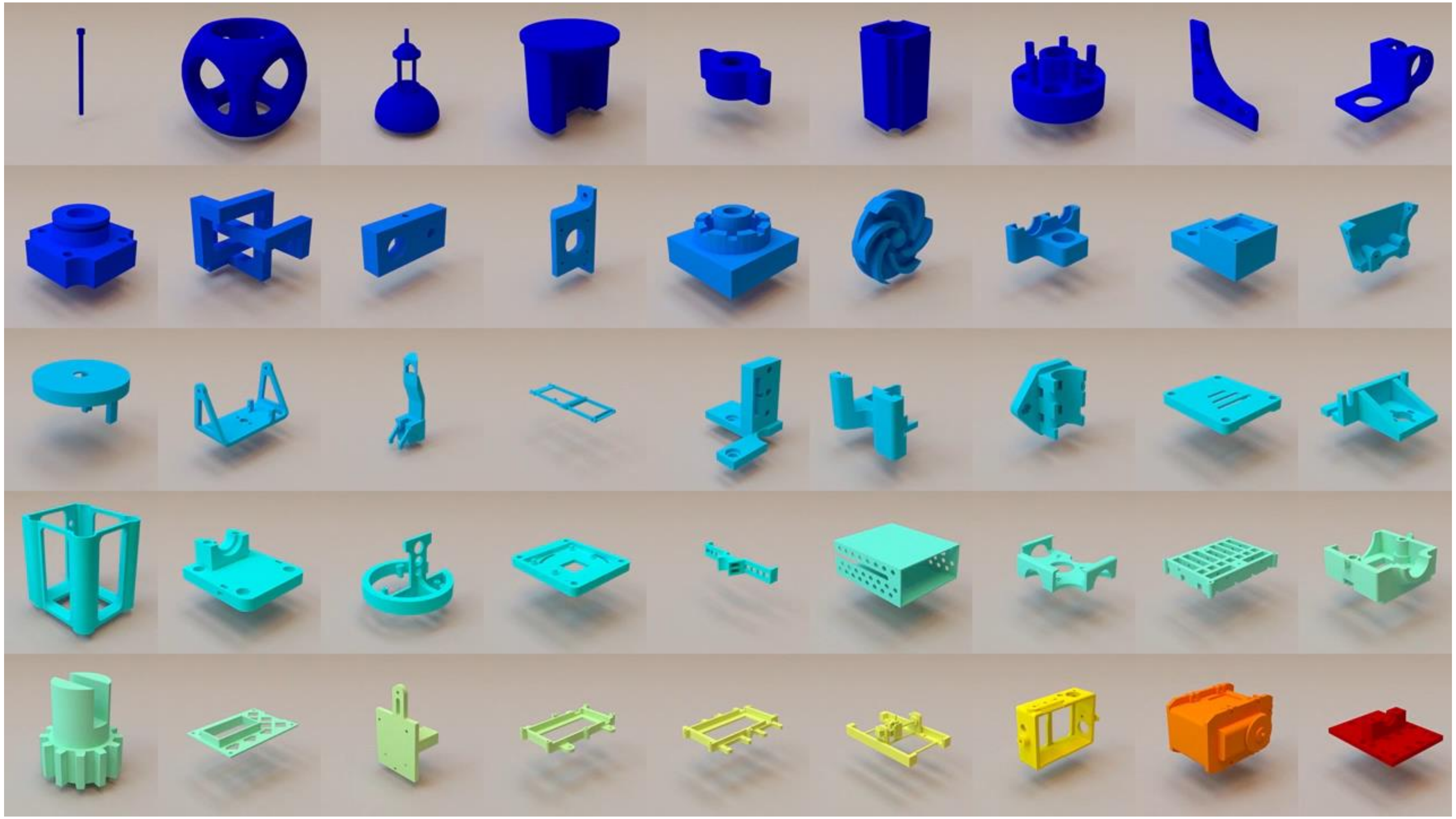
  – Example:

$$O = (s_1 \cap \mathrm{b}_1) - (\mathrm{c}_3 \cup (\mathrm{c}_1 \cup \mathrm{c}_2))$$



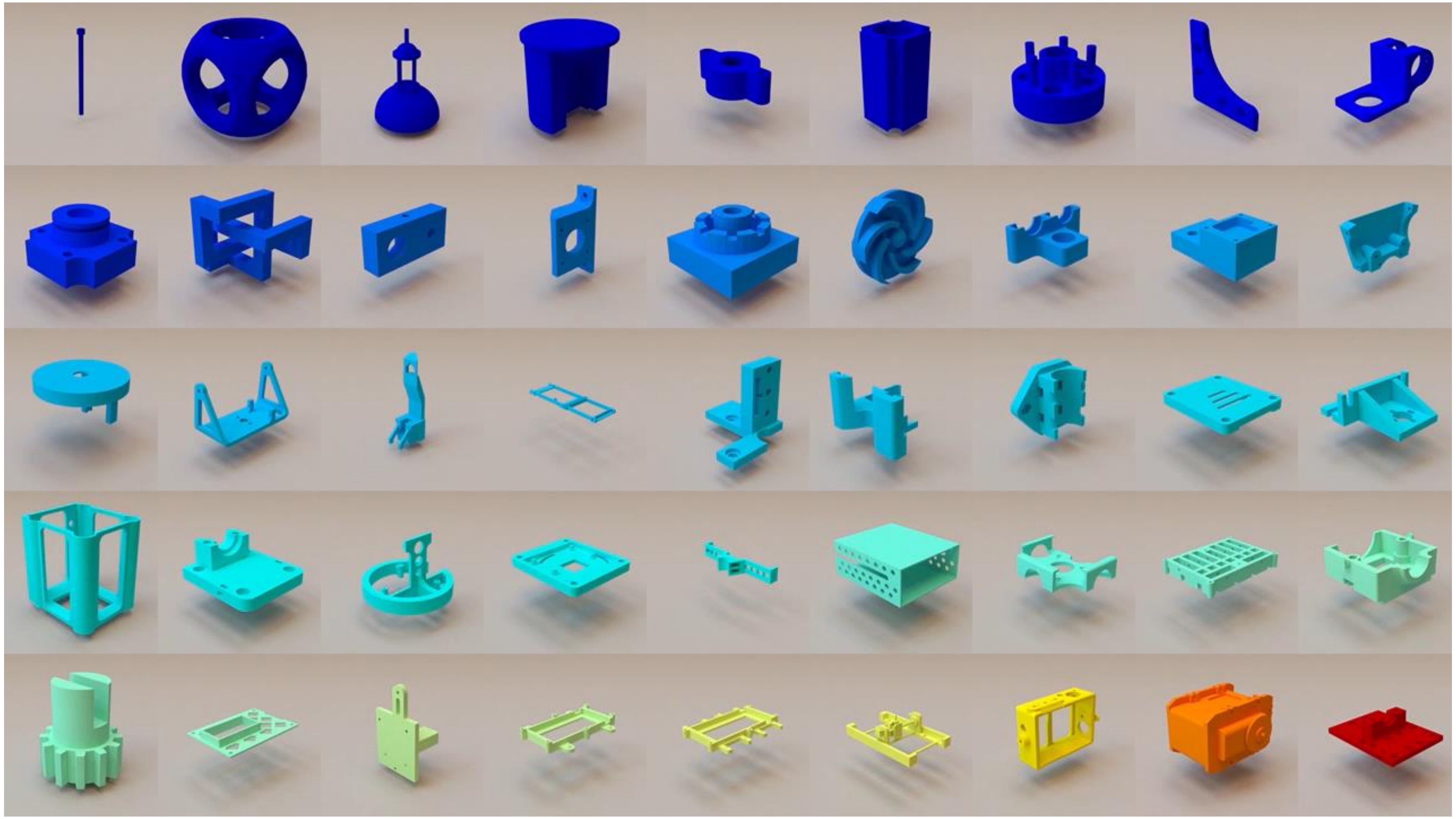
  – Used in design and construction
    - Hard to model “organic” shapes

Constructive Solid Geometry



[Du’18] T.Du et al., “InverseCSG: Automatic Conversion of 3D Models to CSG Trees“

Constructive Solid Geometry



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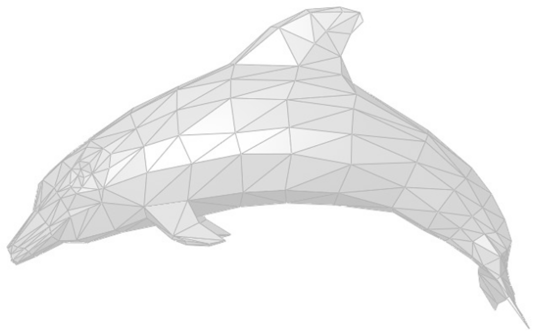

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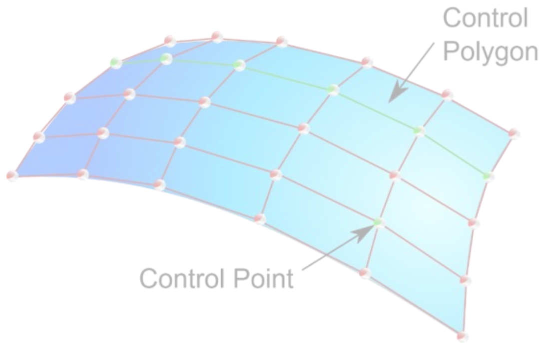
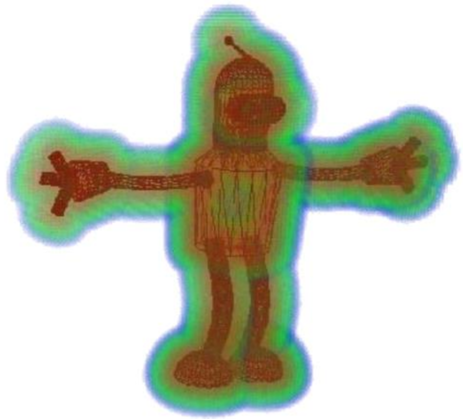


- Implicit Surfaces



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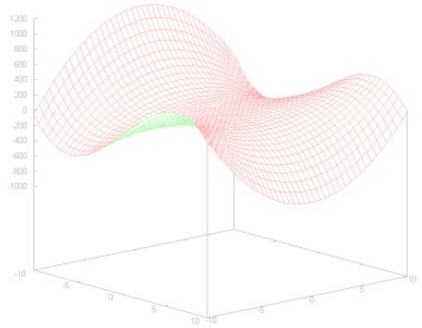


- Constructive Solid Geometry

Implicit Surfaces

- Remember, mathematically:
  - Explicit function: $f(x) = y$
  - Implicit function: $x^2 + y^2 - 1 = 0$

Implicit Surfaces

- [Curless and Levoy], KinectFusion, VoxelHashing, …

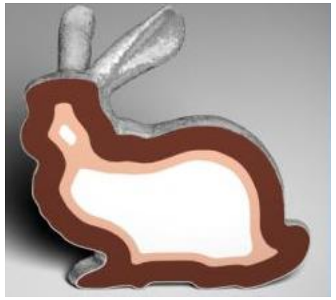
Implicit Surfaces

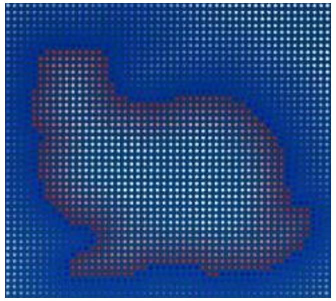
- Implicit form:

$$f(x, y, z)\colon \mathbb{R}^3 \to \mathbb{R}$$

- Surface is defined by the level set of the tri-variate scalar function

$$f(x, y, z) = c$$





Implicit Surfaces

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$$f(x, y, z): \mathbb{R}^3 \rightarrow \mathbb{R}$$

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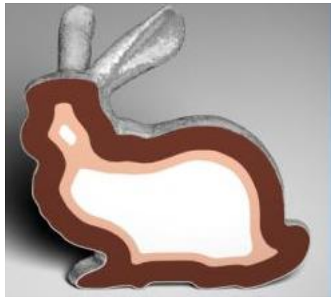
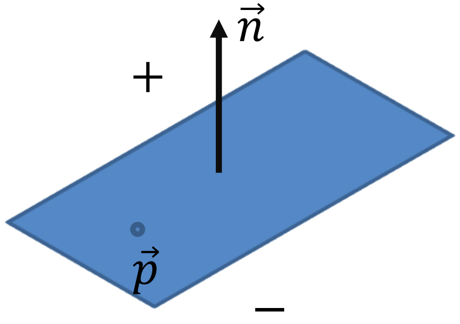
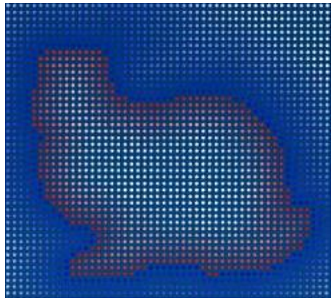


- Example: Hesse normal form

$$f(x, y, z) = \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \vec{p} \right) \cdot \vec{n} = 0$$





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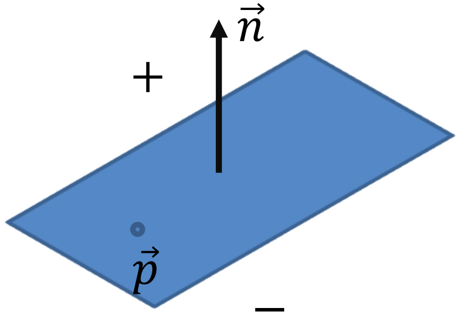
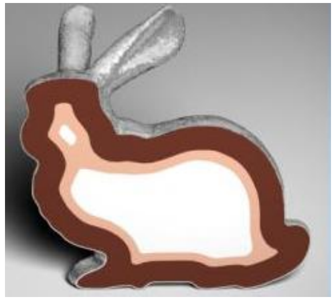
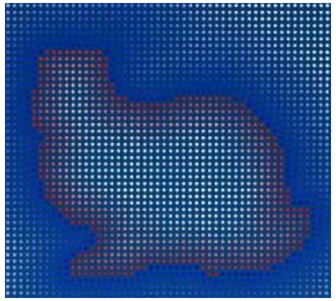


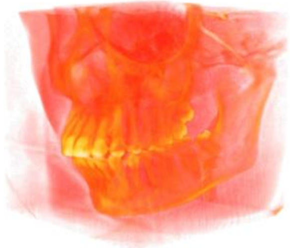


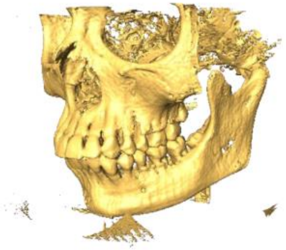


f is also called **Signed Distance Function** (SDF)

Implicit Surfaces

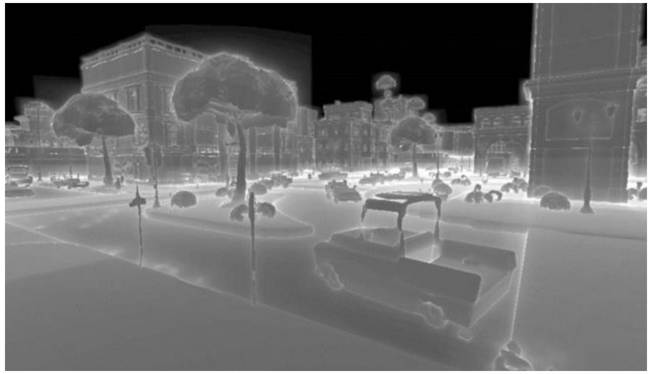
- Examples of such scalar functions $f(x, y, z)$
  - Density



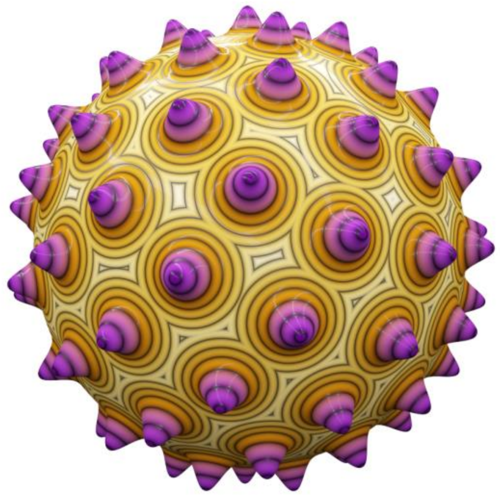


  - Heat

  - Distance

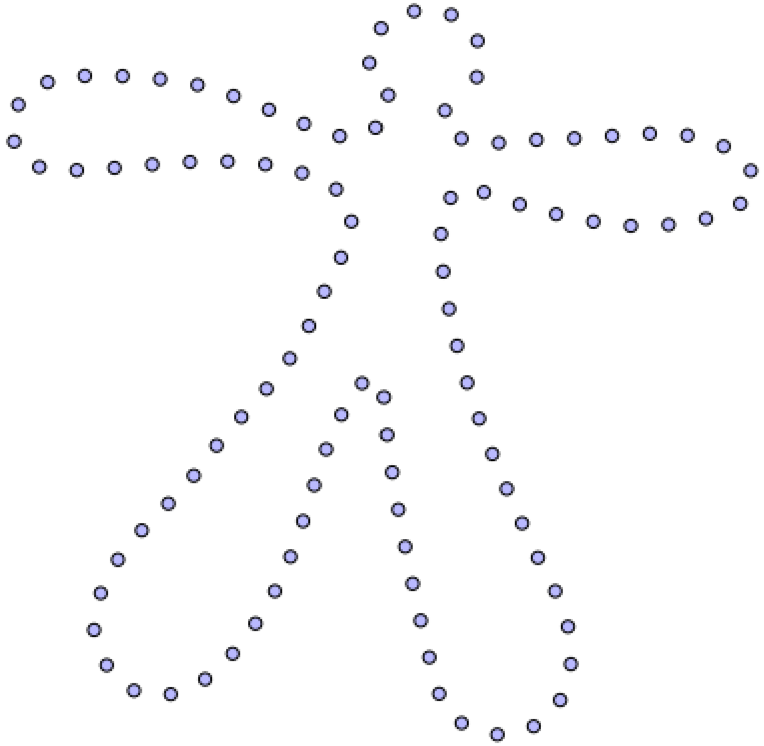


Used for soft shadows in game engines (e.g., Unreal Engine)



Implicit Surfaces

- How to find such a scalar function $f(x, y, z)$?
  - Given sample points



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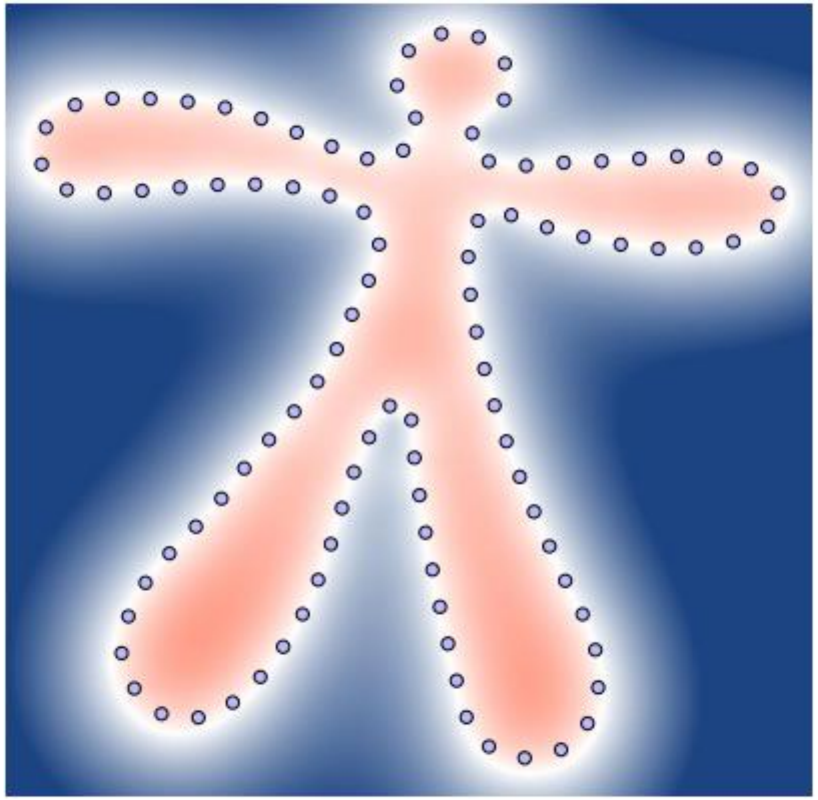
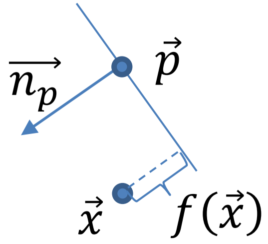


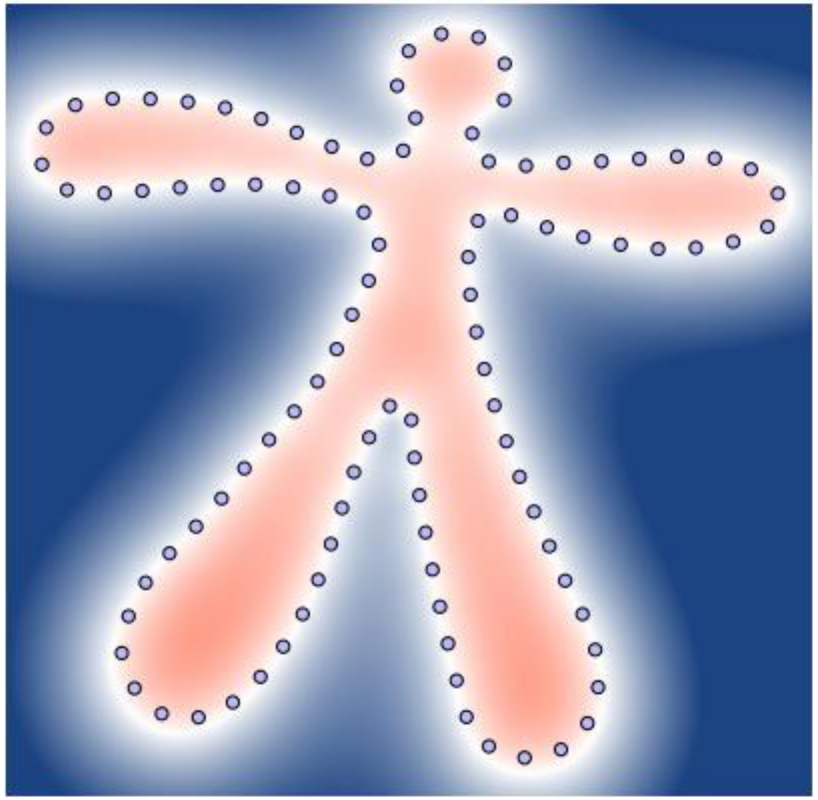
Implicit Surfaces

- How to find such a scalar function $f(x, y, z)$?
  - Hoppe'92:

$$f(\vec{x}) = (\vec{x} - \vec{p}) \cdot \overrightarrow{n_p}$$

$\vec{p}$ is the closest point to $\vec{x}$





[Hoppe'92] H.Hoppe et al., "Surface reconstruction from unorganized points" (SIGGRAPH 92)

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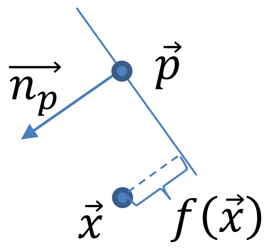

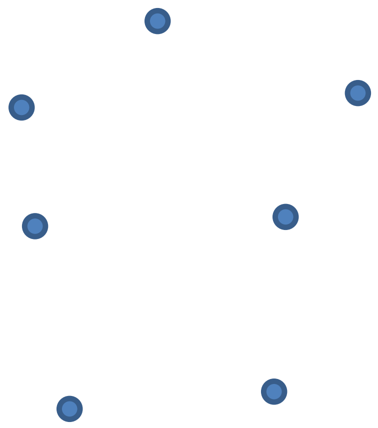

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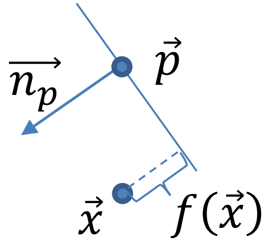

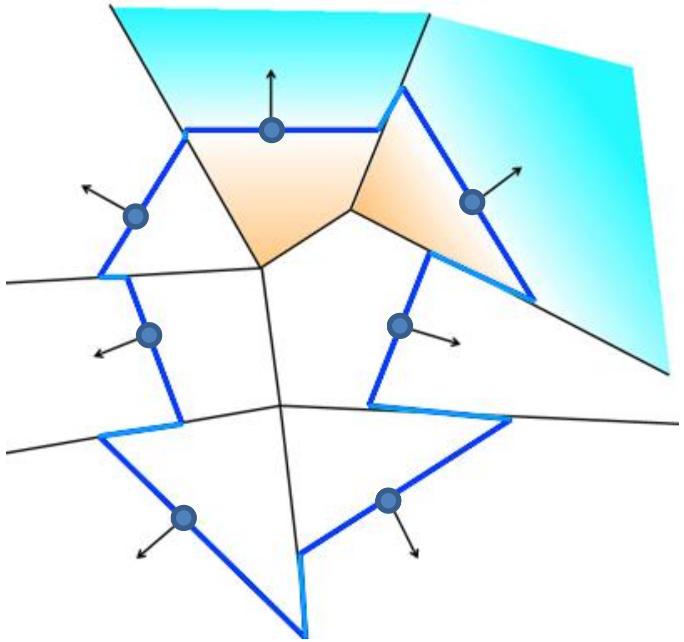

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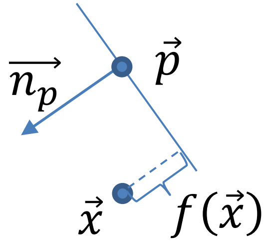

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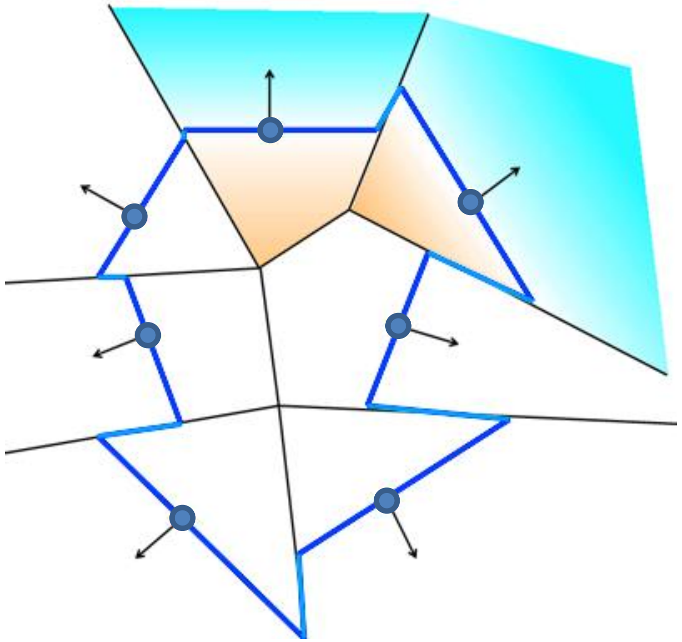
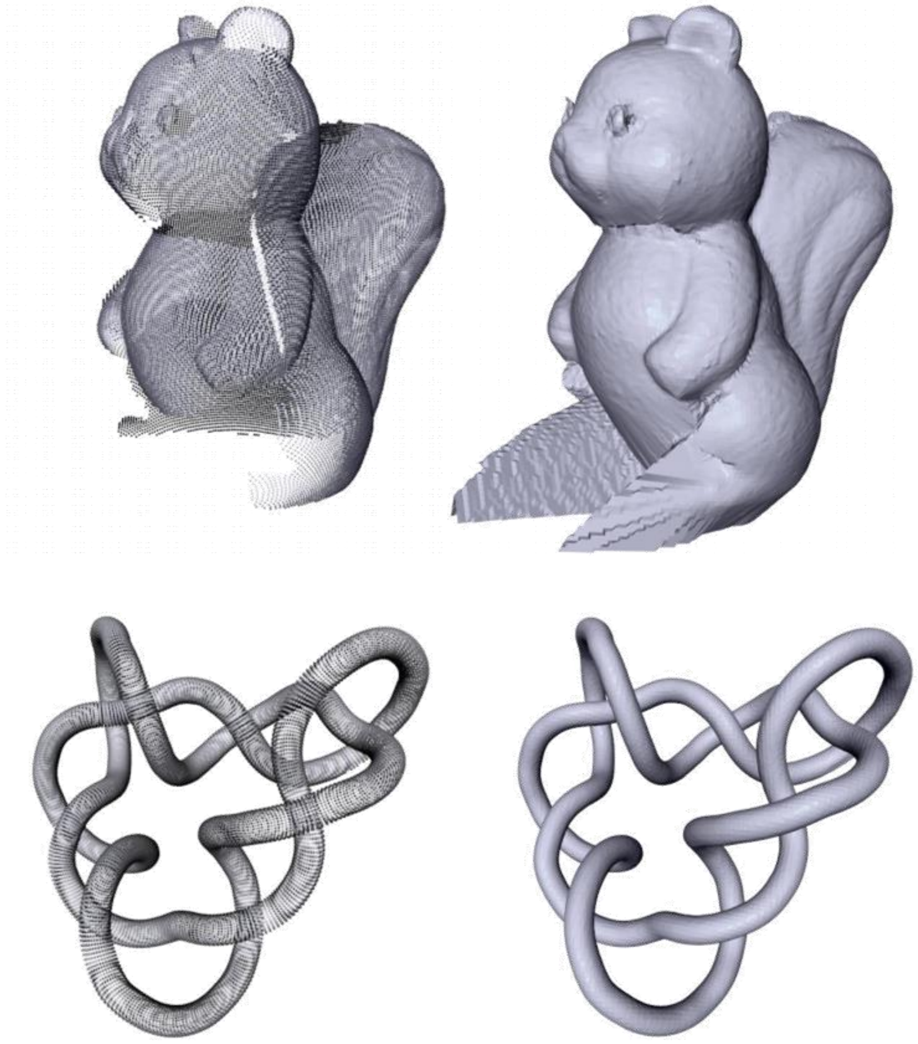


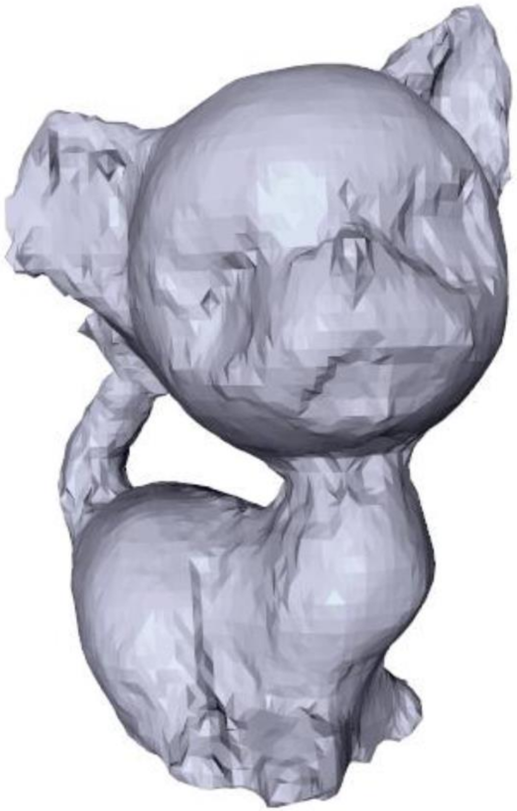
- Piecewise linear, defined on the Voronoi diagram of the input
- Discontinuous along Voronoi edges
- Dependent on the input density

[Hoppe'92] H.Hoppe et al., "Surface reconstruction from unorganized points" (SIGGRAPH 92)

Implicit Surfaces

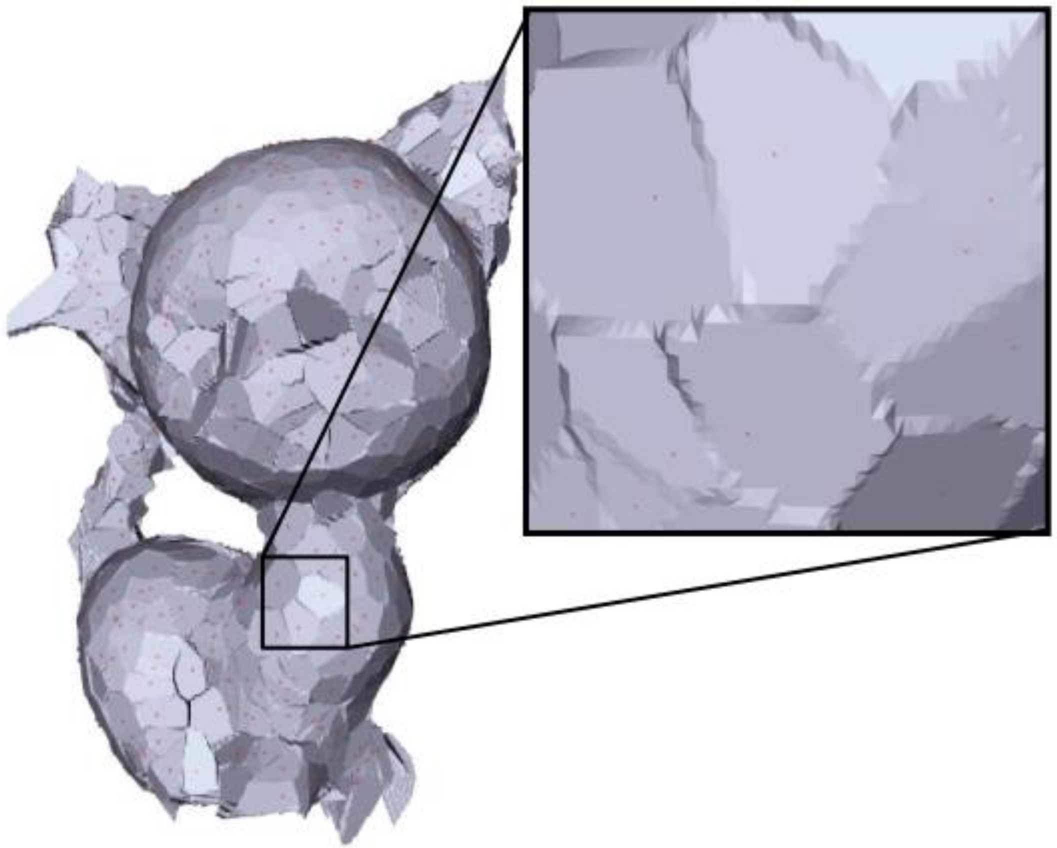


Implicit Surfaces



Piecewise linear surface approximation.

Implicit Surfaces



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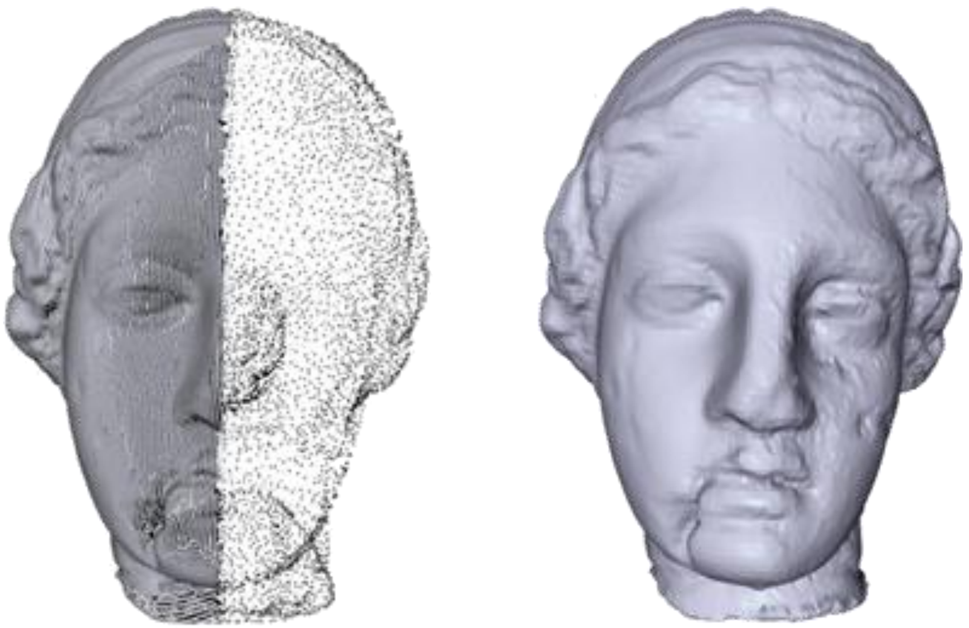
Implicit Surfaces



Hoppe'92 is dependent on the input density

Implicit Surfaces

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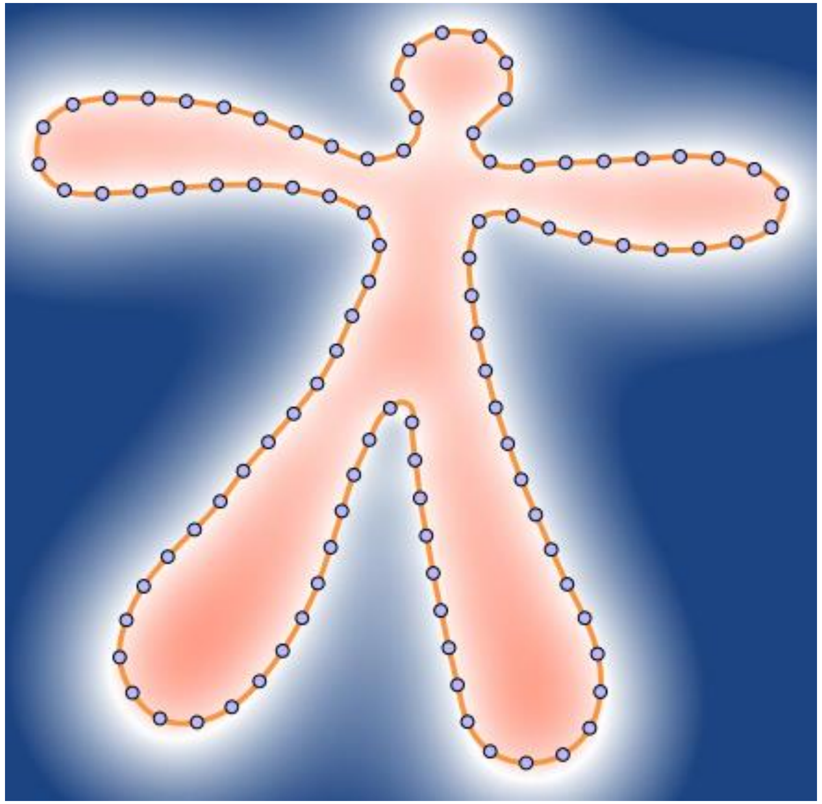
    - $f = 0$ at the sample points
    - $f < 0$ inside the object
    - $f > 0$ outside the object
    - $\boldsymbol{f}$ **should be smooth**



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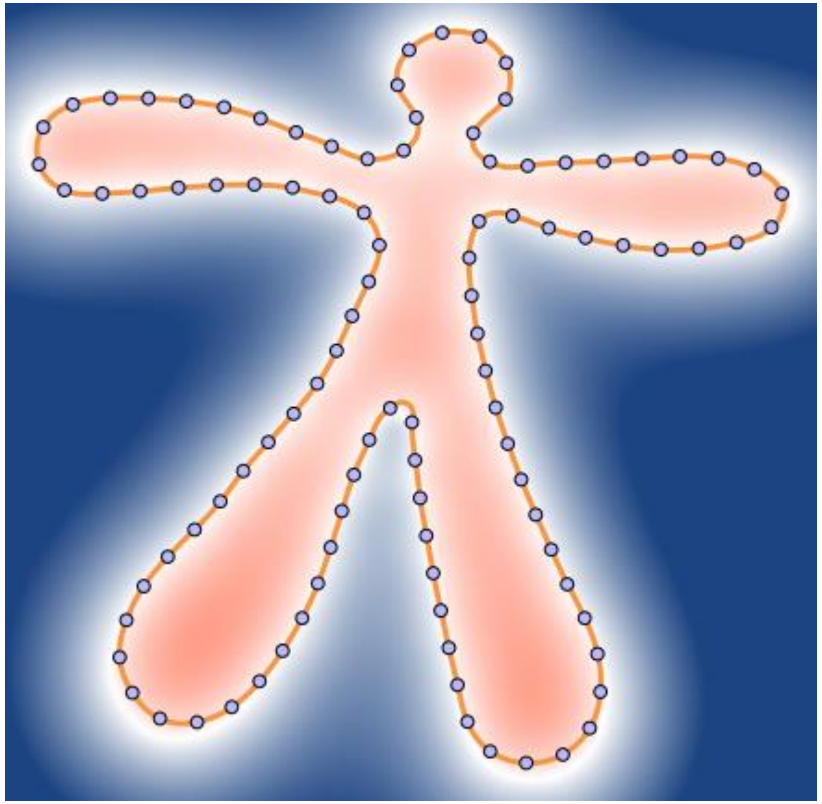
→ **Radial Basis Functions (RBF)**

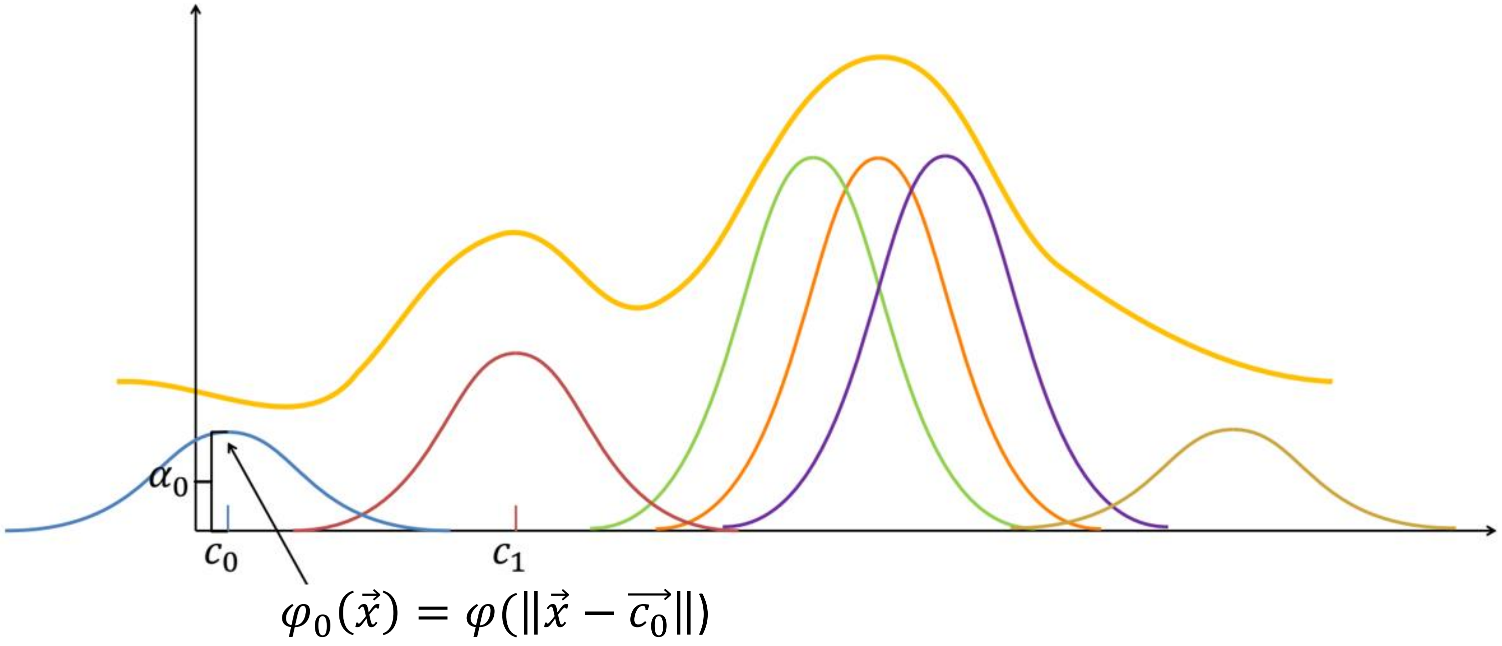


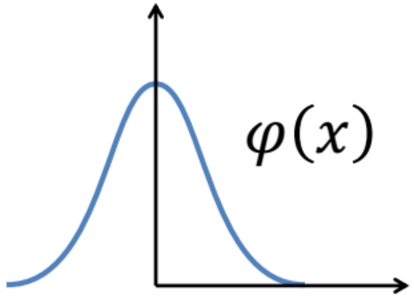
[Turk'99] G.Turk et al., "Variational Implicit Surfaces"

Implicit Surfaces – Radial Basis Functions

- Idea: each complex function can be approximated as the sum of simple scaled and translated kernel functions $\varphi(x)$:

$$g(\vec{x}) \approx f(\vec{x}) = \sum_i \alpha_i \cdot \varphi_i(\vec{x}) = \sum_i \alpha_i \cdot \varphi(\|\vec{x} - \vec{c_i}\|)$$





Implicit Surfaces – Radial Basis Functions

- A **Radial Basis Function** (RBF), also called a **Radial Basis Function Network** is defined as sum of translated and scaled kernels and a linear polynomial:

$$f(\vec{x}) = \sum_i \alpha_i \cdot \varphi_i(\vec{x}) + \underbrace{\vec{b} \cdot \vec{x} + d}_{\text{linear term}}$$

  - Allow for functions of arbitrary complexity!
    - complexity increases with the number of used kernel functions
  - If basis functions $\varphi_i(x)$ are smooth, $f(\vec{x})$ is smooth as well.

Implicit Surfaces – Radial Basis Functions

- Radial Basis Functions in 3D:

$$f(\vec{x}) = \sum_i \alpha_i \cdot \varphi_i(\vec{x}) + \vec{b} \cdot \vec{x} + d$$

  – Use the biharmonic radial basis function:

$$\varphi_i(\vec{x}) = \|\vec{p_i} - \vec{x}\|^3$$

  – Where $\vec{p_i}$ are the input sample points

$$f(\vec{x}) = \sum_i \alpha_i \cdot \|\vec{p_i} - \vec{x}\|^3 + \vec{b} \cdot \vec{x} + d$$

[Turk'99] G.Turk et al., "Variational Implicit Surfaces"

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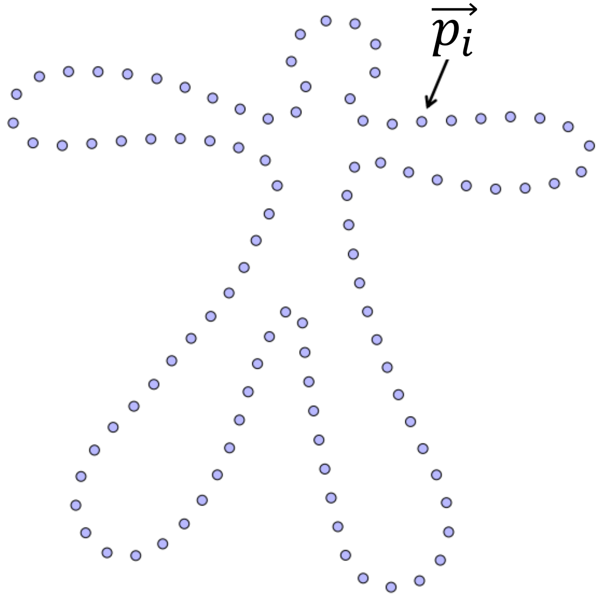

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unknowns

Implicit Surfaces – Radial Basis Functions

$$f(\vec{x}) = \sum_i \underbrace{\boldsymbol{\alpha_i}}_{} \cdot \|\vec{p_i} - \vec{x}\|^3 + \vec{\mathbf{b}} \cdot \vec{x} + \boldsymbol{d}$$

unknowns

- $f(\vec{x})$ should be zero at the sample points:
  - $f(\vec{p_i}) = 0 \ \rightarrow \ n$ equations   (n: number of sample points)

→Solve system of linear equations: $A\vec{x} = \vec{b}$

- Note: system is **underdetermined** since we have $n + 4$ unknowns
- Solving the system will result in the trivial result ($\vec{x} = \vec{0}$)

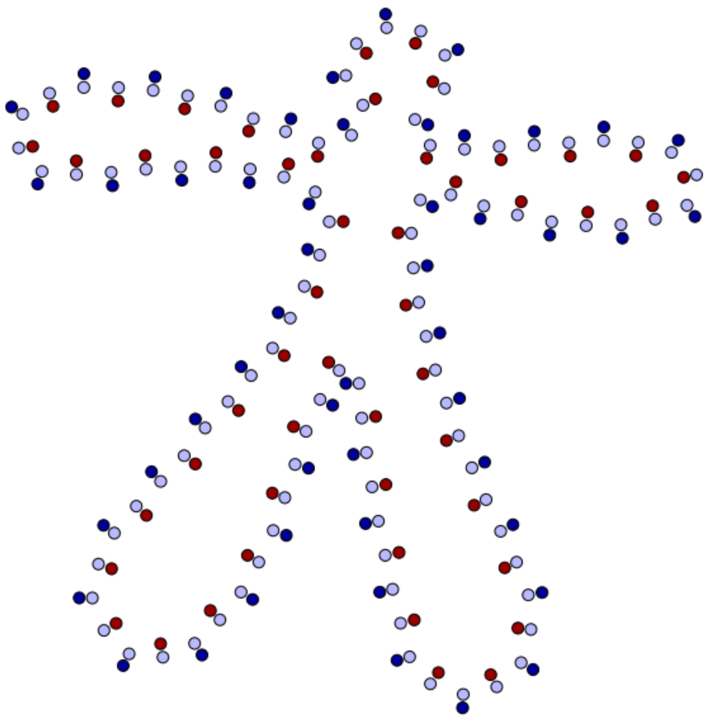
Implicit Surfaces – Radial Basis Functions

$$f(\vec{x}) = \sum_i \boldsymbol{\alpha_i} \cdot \|\vec{p_i} - \vec{x}\|^3 + \vec{\mathbf{b}} \cdot \vec{x} + \boldsymbol{d}$$

unknowns

- $f(\vec{x})$ should be zero at the sample points:
  - $f(\vec{p_i}) = 0 \;\rightarrow\; n$ equations
- Additional constraints where f is non-zero
  - Normal constraints:

    For each sample point add off-surface points by moving the points a little in $\pm$normal direction.
    Set the target distance value of these points to $\pm\epsilon$.



Implicit Surfaces – Radial Basis Functions

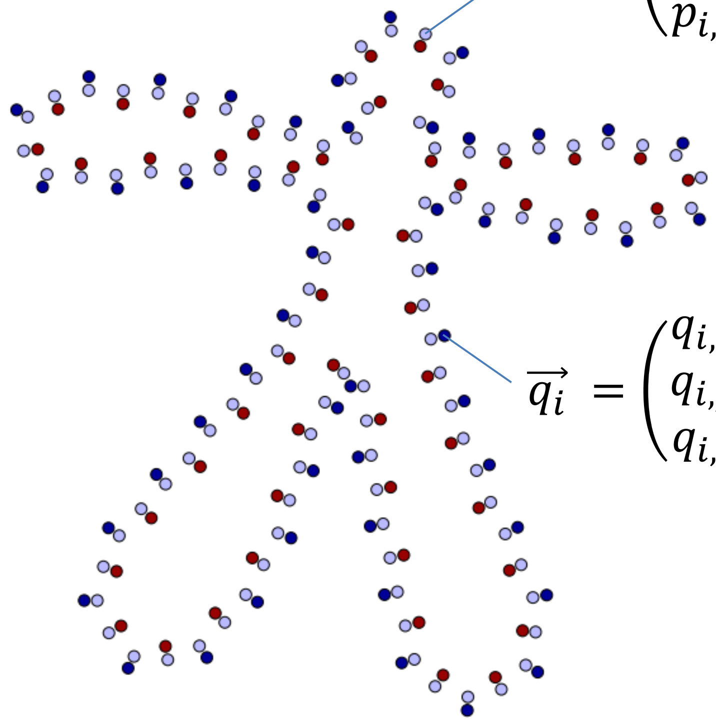
$$f(\vec{x}) = \sum_i \boldsymbol{\alpha_i} \cdot \|\vec{p_i} - \vec{x}\|^3 + \vec{\mathbf{b}} \cdot \vec{x} + \boldsymbol{d}$$

$$\begin{array}{c}
\text{on surface points} \\ \\ \\ \text{off surface points}
\end{array}
\underbrace{\begin{bmatrix}
\varphi_{1,1} & \cdots & \varphi_{1,\mathrm{n}} & p_{1,x} & p_{1,y} & p_{1,z} & 1 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\varphi_{\mathrm{n},1} & \cdots & \varphi_{\mathrm{n},\mathrm{n}} & p_{n,x} & p_{n,y} & p_{n,z} & 1 \\
\hat{\varphi}_{1,1} & \cdots & \hat{\varphi}_{1,\mathrm{n}} & q_{1,x} & q_{1,y} & q_{1,z} & 1 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hat{\varphi}_{\mathrm{n},1} & \cdots & \hat{\varphi}_{\mathrm{n},\mathrm{n}} & q_{n,x} & q_{n,y} & q_{n,z} & 1
\end{bmatrix}}_{A}
\cdot
\underbrace{\begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \\ b_1 \\ b_2 \\ b_3 \\ d \end{bmatrix}}_{\vec{x}}
=
\underbrace{\begin{bmatrix} 0 \\ \vdots \\ 0 \\ \pm\varepsilon \\ \vdots \\ \pm\varepsilon \end{bmatrix}}_{\vec{b}}$$

$$\varphi_{i,j} = \|\vec{p_i} - \vec{p_j}\|^3$$

$$\hat{\varphi}_{i,j} = \|\vec{q_i} - \vec{p_j}\|^3$$

$$\vec{p_i} = \begin{pmatrix} p_{i,x} \\ p_{i,y} \\ p_{i,z} \end{pmatrix}$$

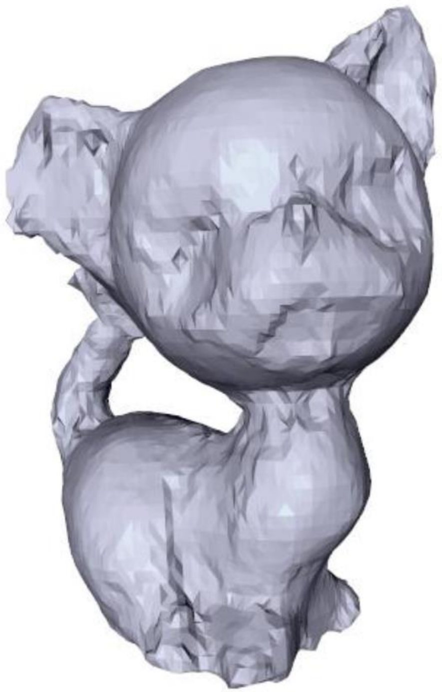


$$\vec{q_i} = \begin{pmatrix} q_{i,x} \\ q_{i,y} \\ q_{i,z} \end{pmatrix}$$

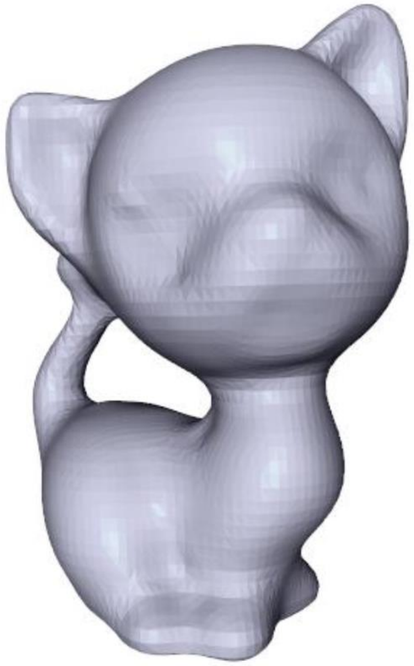
Note: System is **overdetermined**! → use least squares solution ($A^T A \cdot \vec{x} = A^T \vec{b}$)

Implicit Surfaces – Radial Basis Functions

Input Points

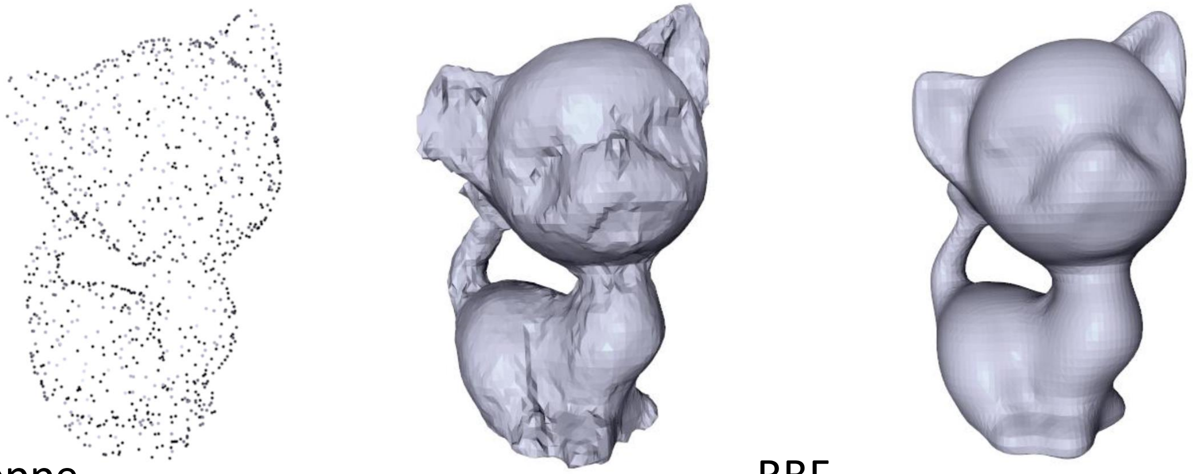


Hoppe



RBF

Implicit Surfaces – Radial Basis Functions



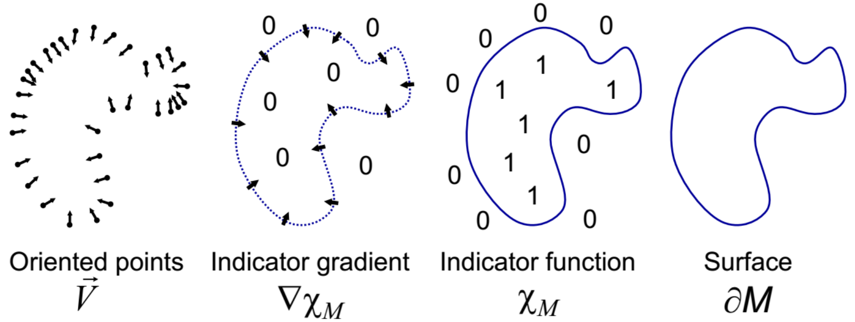
Hoppe
- Local method
- Fast and easy to implement
- Cannot handle noise, outliers, large holes

RBF
- Global method
- Requires solving a linear system (slow)
- Can only handle small point sets
- Can handle noise, outliers

➡ Find a compromise between local and global fitting
- Fit point cloud in a coarse-to-fine manner
- Segment point cloud into parts, fit individually ….

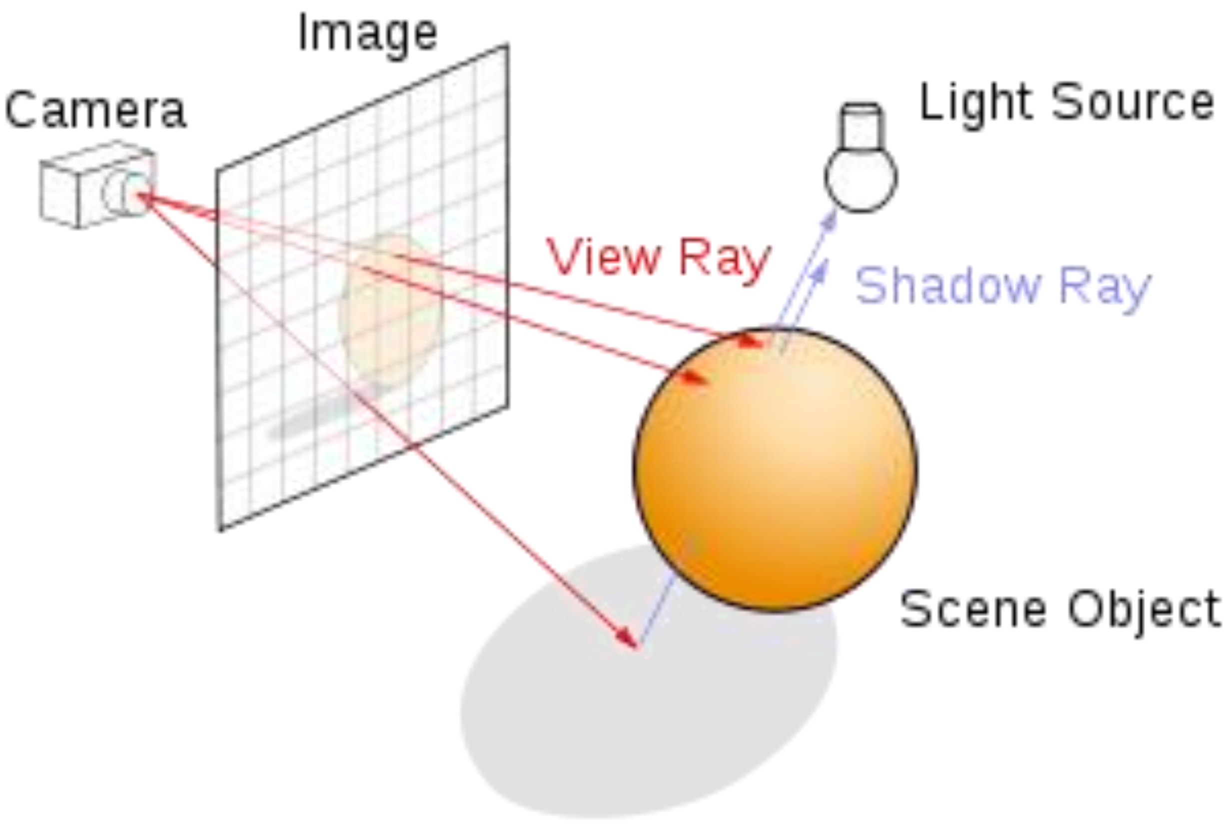
Implicit Surfaces

- Examples of further methods:
  - RBF with floating centers
    - Reduce number of RBF centers
    - Also optimize for the center positions → non-linear
    - [Süßmuth'10] J.Süßmuth "Surface Reconstruction based on Hierarchical Floating Radial Basis Functions"
  - Poisson Reconstruction
    - [Kazhdan'06] M.Kazhdan "Poisson Surface Reconstruction"



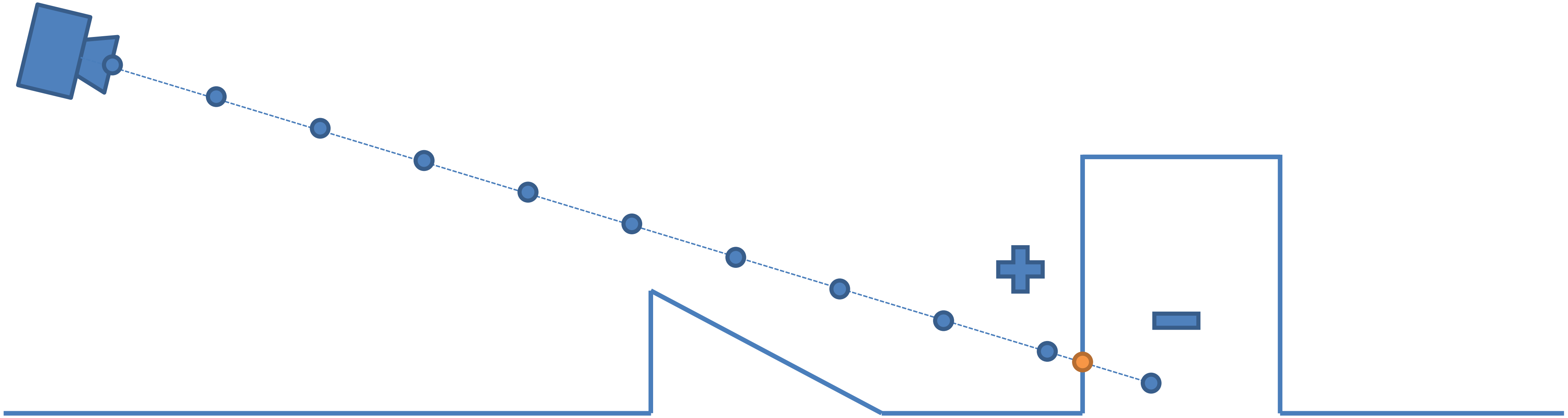
$$\Delta\chi \equiv \nabla \cdot \nabla\chi = \nabla \cdot \vec{V}$$

How to render Signed Distance Functions?



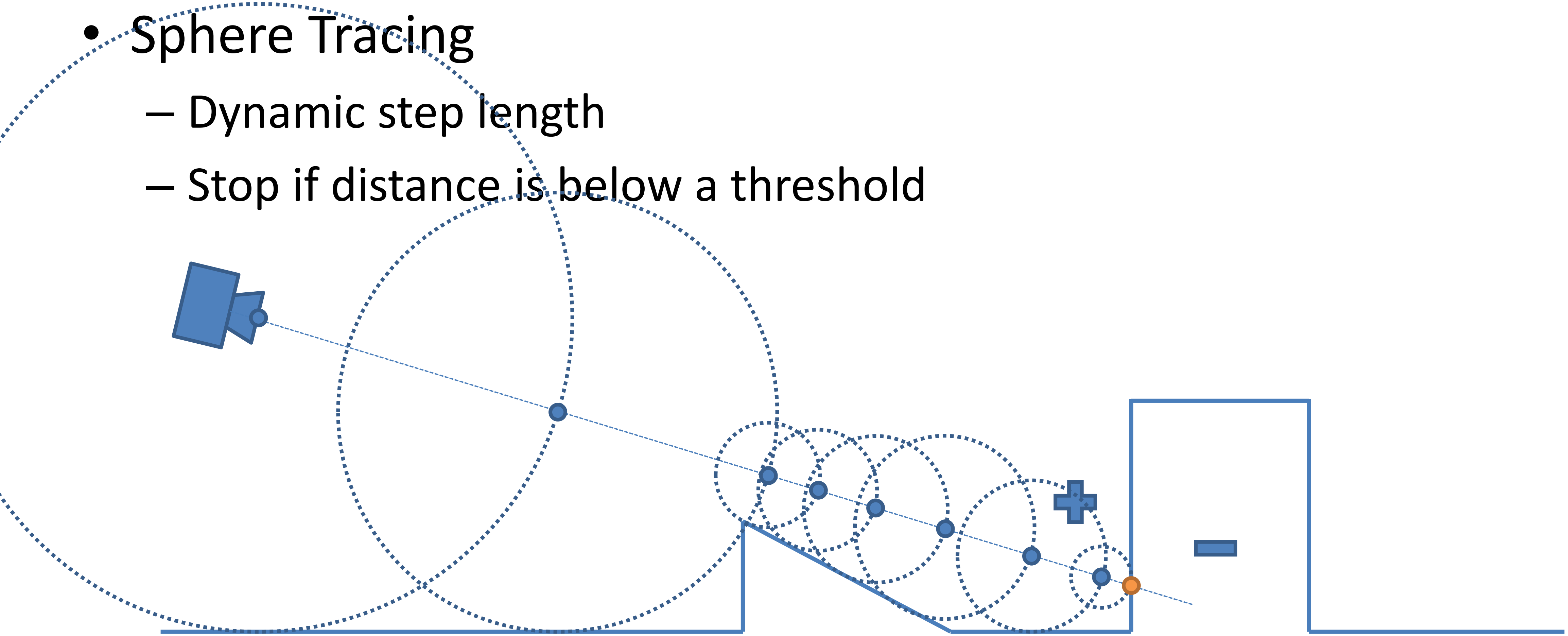
How to render Signed Distance Functions?

- Ray Marching
  - Fixed step length
  - Linear Interpolation, if zero crossing occurs



How to render Signed Distance Functions?

- Sphere Tracing
  - Dynamic step length
  - Stop if distance is below a threshold



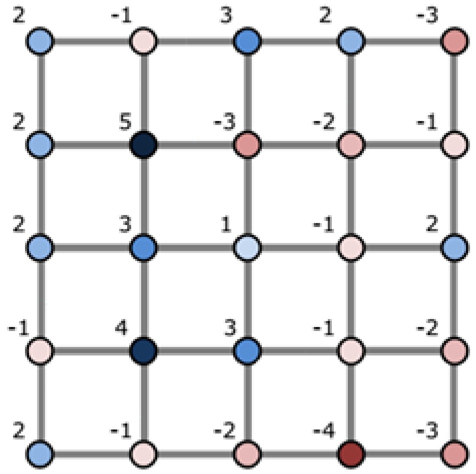
How to render Signed Distance Functions?

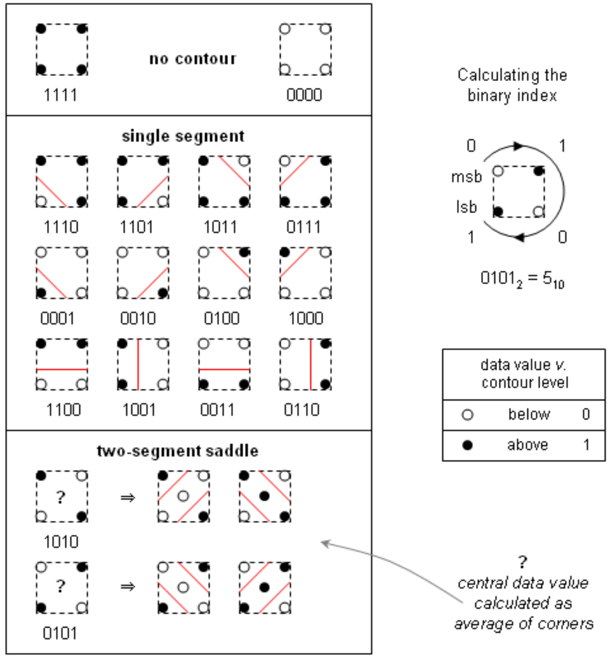
- Convert the iso-surface to polygonal mesh
  - Easy to render
  - Can be used by other post processing pipelines

Marching Cubes

Marching Squares (2D)

- Converts an iso-line of a bi-variat scalar function to a polygon
  - Given: A uniform sampling (on a 2D grid) of the implicit function
  - For every grid cell determine the zero crossings
    - There are $2^4 = 16$ possible combinations





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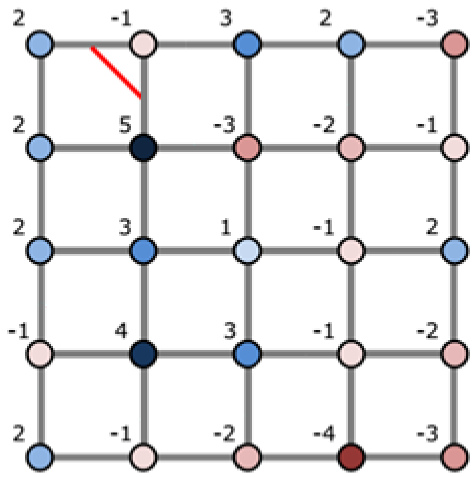

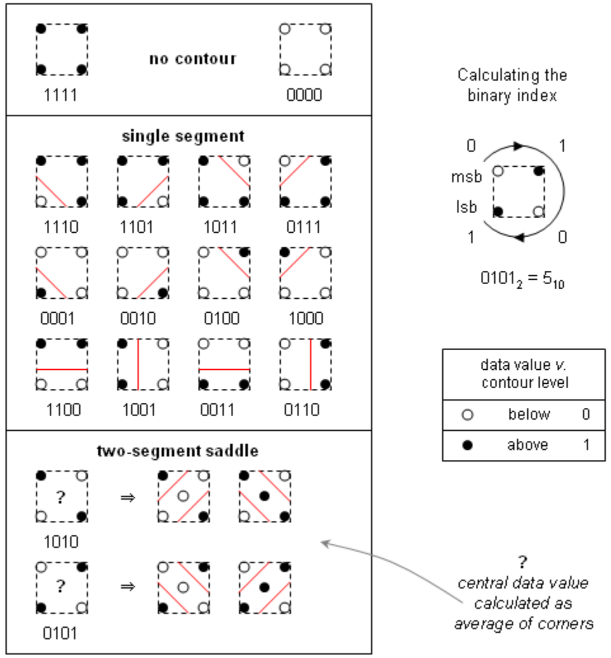

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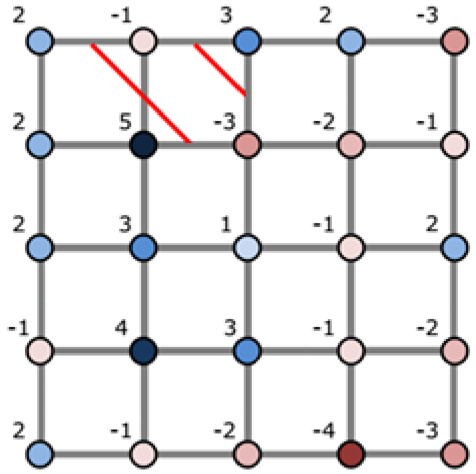

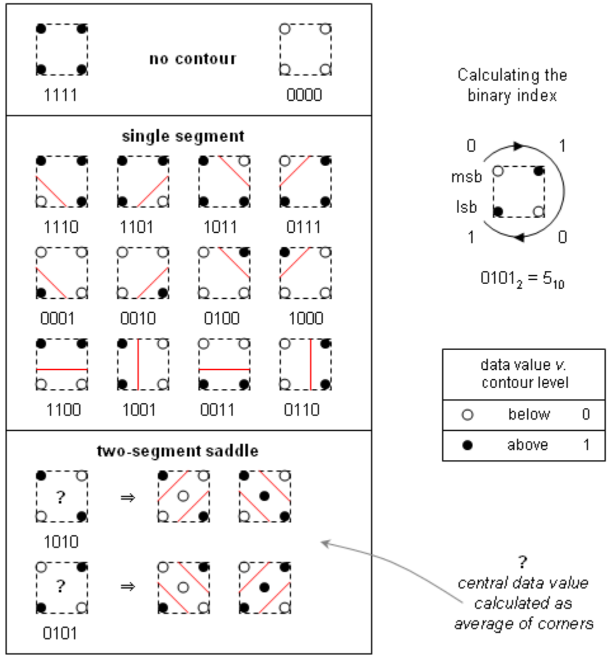

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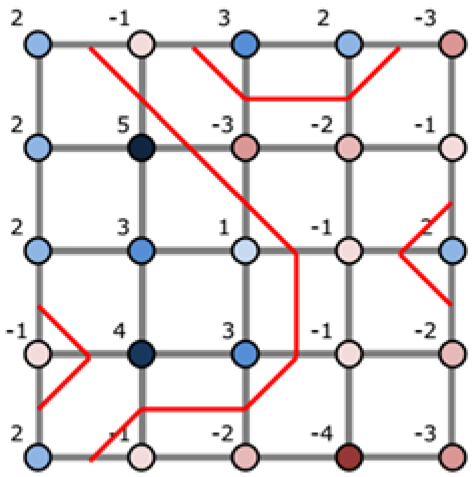
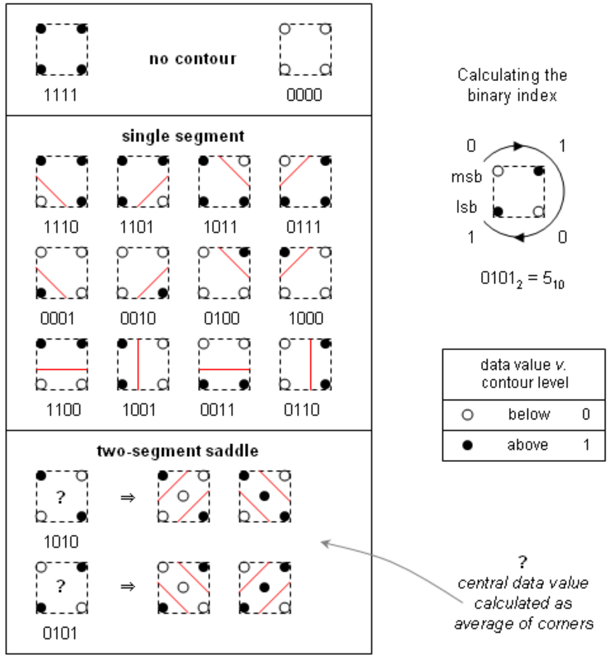




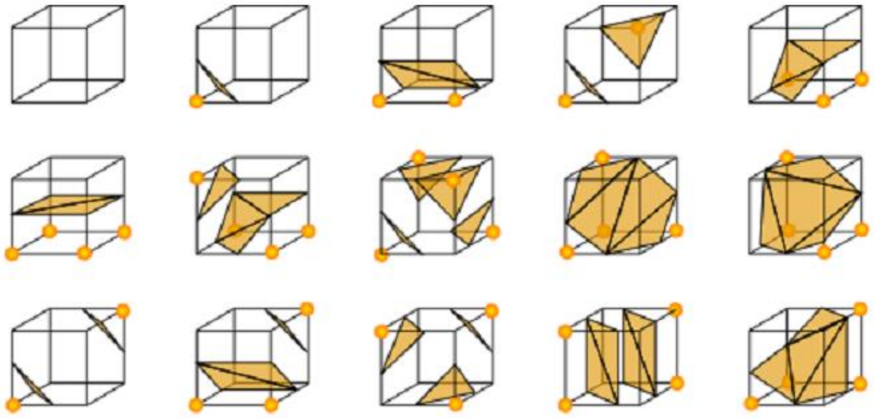
Marching Cubes (3D)

- Converts an iso-surface of a tri-variat scalar function to a polygonal mesh
  - Given: A uniform sampling (on a 3D grid) of the implicit function
  - For every grid cell determine the zero crossings
    - There are $2^8 = 256$ possible combinations
    - Use lookup table to find triangulation
    - Adjust vertex position according to linear interpolation

[Lorensen’87] W.Lorensen et al., “Marching cubes: A high resolution 3D surface construction algorithm“

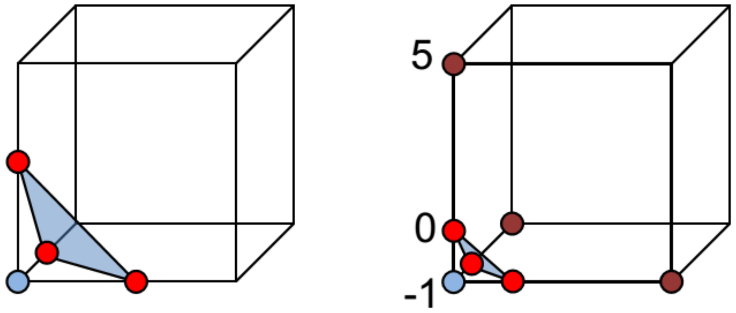
Marching Cubes (3D)

- Lookup table:



Marching Cubes table

- Linear interpolation:



From table Real intersections

Marching Cubes (3D)

```cpp


//////////////////////////////////////////////////////
// tables
//////////////////////////////////////////////////////

// Polygonising a scalar field
// Also known as: "3D Contouring", "Marching Cubes", "Surface Reconstruction"
// Written by Paul Bourke
// May 1994
// http://paulbourke.net/geometry/polygonise/


const static int edgeTable[256] = {
        0x0, 0x109, 0x203, 0x30a, 0x406, 0x50f, 0x605, 0x70c,
        0x80c, 0x905, 0xa0f, 0xb06, 0xc0a, 0xd03, 0xe09, 0xf00,
        0x190, 0x99, 0x393, 0x29a, 0x596, 0x49f, 0x795, 0x69c,
        0x99c, 0x895, 0xb9f, 0xa96, 0xd9a, 0xc93, 0xf99, 0xe90,
        0x230, 0x339, 0x33, 0x13a, 0x636, 0x73f, 0x435, 0x53c,
        0xa3c, 0xb35, 0x83f, 0x936, 0xe3a, 0xf33, 0xc39, 0xd30,
        0x3a0, 0x2a9, 0x1a3, 0xaa, 0x7a6, 0x6af, 0x5a5, 0x4ac,
        0xbac, 0xaa5, 0x9af, 0x8a6, 0xfaa, 0xea3, 0xda9, 0xca0,
        0x460, 0x569, 0x663, 0x76a, 0x66, 0x16f, 0x265, 0x36c,
        0xc6c, 0xd65, 0xe6f, 0xf66, 0x86a, 0x963, 0xa69, 0xb60,
        0x5f0, 0x4f9, 0x7f3, 0x6fa, 0x1f6, 0xff, 0x3f5, 0x2fc,
        0xdfc, 0xcf5, 0xfff, 0xef6, 0x9fa, 0x8f3, 0xbf9, 0xaf0,
        0x650, 0x759, 0x453, 0x55a, 0x256, 0x35f, 0x55, 0x15c,
        0xe5c, 0xf55, 0xc5f, 0xd56, 0xa5a, 0xb53, 0x859, 0x950,
        0x7c0, 0x6c9, 0x5c3, 0x4ca, 0x3c6, 0x2cf, 0x1c5, 0xcc,
        0xfcc, 0xec5, 0xdcf, 0xcc6, 0xbca, 0xac3, 0x9c9, 0x8c0,
        0x8c0, 0x9c9, 0xac3, 0xbca, 0xcc6, 0xdcf, 0xec5, 0xfcc,
        0xcc, 0x1c5, 0x2cf, 0x3c6, 0x4ca, 0x5c3, 0x6c9, 0x7c0,
        0x950, 0x859, 0xb53, 0xa5a, 0xd56, 0xc5f, 0xf55, 0xe5c,
```

```cpp
const static int triTable[256][16] =
{ { -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 0, 8, 3, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 0, 1, 9, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 1, 8, 3, 9, 8, 1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 1, 2, 10, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 0, 8, 3, 1, 2, 10, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 9, 2, 10, 0, 2, 9, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 2, 8, 3, 2, 10, 8, 10, 9, 8, -1, -1, -1, -1, -1, -1, -1 },
{ 3, 11, 2, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 0, 11, 2, 8, 11, 0, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 1, 9, 0, 2, 3, 11, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 1, 11, 2, 1, 9, 11, 9, 8, 11, -1, -1, -1, -1, -1, -1, -1 },
{ 3, 10, 1, 11, 10, 3, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 0, 10, 1, 0, 8, 10, 8, 11, 10, -1, -1, -1, -1, -1, -1, -1 },
{ 3, 9, 0, 3, 11, 9, 11, 10, 9, -1, -1, -1, -1, -1, -1, -1 },
{ 9, 8, 10, 10, 8, 11, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 4, 7, 8, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 4, 3, 0, 7, 3, 4, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 0, 1, 9, 8, 4, 7, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 4, 1, 9, 4, 7, 1, 7, 3, 1, -1, -1, -1, -1, -1, -1, -1 },
{ 1, 2, 10, 8, 4, 7, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 3, 4, 7, 3, 0, 4, 1, 2, 10, -1, -1, -1, -1, -1, -1, -1 },
{ 9, 2, 10, 9, 0, 2, 8, 4, 7, -1, -1, -1, -1, -1, -1, -1 },
{ 2, 10, 9, 2, 9, 7, 2, 7, 3, 7, 9, 4, -1, -1, -1, -1 },
{ 8, 4, 7, 3, 11, 2, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
{ 11, 4, 7, 11, 2, 4, 2, 0, 4, -1, -1, -1, -1, -1, -1, -1 },
{ 9, 0, 1, 8, 4, 7, 2, 3, 11, -1, -1, -1, -1, -1, -1, -1 },
{ 4, 7, 11, 9, 4, 11, 9, 11, 2, 9, 2, 1, -1, -1, -1, -1 },
{ 3, 10, 1, 3, 11, 10, 7, 8, 4, -1, -1, -1, -1, -1, -1, -1 },
{ 1, 11, 10, 1, 4, 11, 1, 0, 4, 7, 11, 4, -1, -1, -1, -1 },
{ 4, 7, 8, 9, 0, 11, 9, 11, 10, 11, 0, 3, -1, -1, -1, -1 },
{ 4, 7, 11, 4, 11, 9, 9, 11, 10, -1, -1, -1, -1, -1, -1, -1 },
{ 9, 5, 4, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1 },
```

https://github.com/angeladai/VirtualScan/blob/master/src/Tables.h

Administrative

Reading Homework:

- Poisson Surface Reconstruction [Kazhdan et al.]
  https://hhoppe.com/poissonrecon.pdf
- Screened Poisson Surface Reconstruction [Kazhdan and Hoppe]
  https://www.cs.jhu.edu/~misha/MyPapers/ToG13.pdf

Next week:

- Overview of 3D reconstruction methods

Administrative

See you next week!